Co-integrating currencies and yield differentials

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Received 27 January 2004; received in revised form 16 August 2004; accepted 15 April 2005  

Available online 18 July 2005

Abstract

This study investigates the relationship between currencies and interest rates of different maturity horizons. The real exchange rate is found to depend both on short-term real domestic and foreign interest rate difference and on long-term real domestic and foreign interest rate difference. Co-integrating regressions of contemporaneous currency rates generate negative and significant coefficients for long-term rate differentials, consistent with uncovered interest parity. Therefore, the expectations hypothesis holds for long horizons. On the other hand, positive coefficients for real short-term interest rate differentials reveal the forward premium puzzle: the failure of uncovered interest parity for short-horizons. Results are partly driven by the very different risk characteristics of short-term bonds and foreign bonds.

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JEL classification: F31; E43; G14  

Keywords: Exchange rates; Interest rates; Uncovered interest parity; Forward premium puzzle

1. Introduction

Central banks of governments utilize interest rates for numerous macroeconomic purposes; to stimulate or slow down an economy, to attract foreign capital, to strengthen the domestic currency, to weaken the local currency in order to offset a trade deficit, just to name a few. Rational expectations say that high interest rates are to compensate for future currency depreciation. Empirically, however, the opposite is seen: high interest rate currencies become more valuable. The anomaly has been...
termed the forward premium puzzle (FPP). This paper proposes a simple theoretical framework to account for the puzzle. Empirical investigation shows that the FPP appears in the relationship between short-term rates and the exchange rate. Expectations hold in the relationship between long-term rates and the exchange rate.

Studies in the literature mostly focus on short-term rates which can be changed most effectively by the central monetary authority. However, the longer maturity bond yield reaction to these changes should also be considered. There is not a positive, linear response in long-term yields as short-term rates change. Accordingly, this article considers both the short term and the long term in establishing the relationship between currencies and interest rates.

The framework starts with a portfolio of short- and long-term US government bonds and foreign currency (representing short-term foreign bonds) designed to meet a desired risk level. The long-term bond is more risky than the short-term bond but less risky than foreign bond and can be a substitute to both. In this setting, when the short-term rate is increased by the Federal Reserve, the short-term bond price declines because yields and prices move in opposite directions. The demand for these T-bills increases while the demand for the alternative long-term bonds declines. The decline in demand for long-term bonds increases the demand for the other alternative, foreign currency, and this is reflected to an instant increase in the value of the foreign currency (i.e., an instant depreciation in US dollar). Furthermore, high priced long-term bonds with lower yields will be negatively associated with the instant foreign currency appreciation. In steady state equilibrium, demand functions of domestic bonds will adjust to the shock and lead to an appreciation of the domestic currency, which is the FPP. Similarly, the supply–demand dynamics will work in the reverse direction if short rates are reduced.

The empirical analyses affirm the above theoretical conjectures and reveal, in real terms: (1) the risk ranking of the three assets consistent with theory; (2) a positive relationship between the contemporaneous exchange rate and the short-term domestic and foreign rate differential; (3) a negative relationship between the change in the exchange rate and short-term rate differential; (4) a negative relationship between the contemporaneous exchange rate and the long-term rate differential; (5) a positive relationship between the change in the exchange rate and long-term rate differential.

1.1. Literature

The often tested theory in the literature is the Expectations Hypothesis (EH) or Uncovered Interest Parity (UIP). EH says that the forward/futures rates are unbiased estimates of the expected future spot rates. Therefore, the futures-spot spread, or interest rate difference from covered interest parity, can be used to predict changes in future exchange rates. In other words, if all market participants are risk neutral, they should agree on a forward/futures price that provides an expected profit of zero to all parties. Under free floating exchange rate regimes, capital inflows resulting from interest rate differentials will cause a surplus in the balance of payments. This should cause a temporary appreciation (less than 1 month according to Fama, 1984, and others) of domestic currency but in overall equilibrium, the dollar should depreciate. Thus, according to the expectations hypothesis, high interest rate currencies should lose value in the future. In fact, that is precisely why these currencies offer high rates: to

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1 Many fixed income portfolio decisions are driven by asset allocation; or, investment horizon and duration for immunization. An alternative approach in the literature and in this article is to study supply and demand dynamics of bonds in portfolios. The goal is to maintain a desired risk level as individual assets become more or less attractive.
compensate investors in equilibrium. Empirically, if the change in the exchange rate is regressed on the domestic and foreign interest rate difference, the regression slope coefficient is expected to be positive and equal to one according to EH (which says forward rates are expected future spot rates). However, numerous studies have found the coefficient to be significantly negative. This is the forward premium puzzle (FPP) named by Fama (1984), though it was previously identified by Frankel (1979) and others.

The literature has since moved in several directions. Studies using long-term forecast horizons and long-term interest rates as explanatory variables find results in line with the uncovered interest rate parity. For example, Alexius (1993) and Meredith and Chinn (1998), report evidence of positive slope coefficients for long-term interest rates over long horizons. On the other hand, studies such as those by Hansen and Hodrick (1980) and Hsieh (1993) focus on different financial markets and find that the FPP is extensive in futures/forward markets.

Another branch of studies including Ahn (2004), Backus, Foresi, and Telmer (2001), and Inci and Lu (2004) have developed sophisticated theoretical and stochastic models that are robust to the puzzle with the goal of identifying the factors (local and/or common) that drive economies.

A third group has focused on the empirical models and the statistical methodologies used to address the issue. Baxter (1994), Dornbusch (1976), and Frankel (1979) have proposed that prices move slowly over time in reaction to economic shocks and innovations. This sticky price approach has generated interesting empirical models, one of which is modified and used in this study. Finally, econometricians have developed more complex estimation procedures to analyze the data. Many currency and interest rate data are nonstationary. Such series are defined to have unit roots. Sophisticated co-integrating regression models have been developed to filter the nonstationarity and to establish the actual relationship between currencies and interest rates. These studies have produced mixed results. Meese and Rogoff (1988) find no evidence of a significant relationship between real exchange rates and long-term real interest rate differentials but they do not examine short and long rates simultaneously. Edison and Pauls (1993) have suggested omitted variable(s) in the relationship. Accordingly, Throop (1993) has ascertained a link between exchange rates and interest rates by adding other variables, such as the budget balance and real oil prices, but without a theoretical justification.

Although results are mixed due to differences in estimation techniques and forecast horizons of interest rates, the common conclusion is that the relationship between currency returns and interest rate differentials depends on two key points: the term structure of interest rates and the financial risks embedded in a country as perceived by investors. This paper addresses these points and provides a theoretical conjecture of their effect on exchange rate dynamics. The term structure is constructed with short and long-term rates. Risk is measured by the standard deviation of long-term government bond return and the standard deviation of the exchange rate change, both conditional on time. The standard deviation of short-term domestic government bond return is assumed to be negligible relative to the other two assets.²

The empirical analyses start by justifying the theoretical risk ranking of the assets. Co-integrating regressions of contemporaneous currency rates are then conducted and negative and significant coefficients for long-term interest rate differentials are generated. This is consistent with uncovered interest parity, and therefore, the expectations hypothesis is valid for long horizons. On the other hand, positive coefficients obtained for the real short-term interest rate differentials are consistent with the

² Stochastic models such as that of Dai and Singleton (2000) can be used to include more maturities. However, the simpler approach in this study tackles the issue more intuitively and directly. Also, using government bonds in the theoretical discussion and the empirical tests eliminates additional risks such as the default risk associated with corporate bonds.
forward premium puzzle, and demonstrate the failure of uncovered interest parity for short-horizons. The results are robust to a different regression set where the dependent variable is the future currency depreciation.

In the next section the general theory and conjectures are presented. Section 3 is about the design of empirical tests. Section 4 explains the data and the descriptive statistics. Section 5 presents the results. The conclusions are presented in the final section.

2. General theory

Uncovered interest parity is based on risk neutrality. To have a better understanding of its failure and to establish a more sophisticated relationship between the interest rate difference and currency dynamics, a risk-averse world should be examined. When investors are risk-averse, the term structure of interest rates becomes a significant part of the analysis. Thus, the investor is assumed to have three assets: short-term domestic government bonds, long-term domestic government bonds, and short-term foreign government bonds (so that the change in the exchange rate is the main source of risk). The investor forms a portfolio based on his/her risk tolerance using the three assets. The investor changes the proportions of the three assets as they become more or less attractive, while maintaining the desired risk level. The portfolio update depends on whether these assets are alternatives or complements of each other. Any two assets having similar risk characteristics are termed as substitutes. Demand functions move in opposite directions for such substitutes. On the other hand, two assets with different risk characteristics, i.e., a very risky asset and a near riskless one, are complements for investment purposes. The demand and price functions of these complements move in the same direction.

In a general analysis of a portfolio of three securities, each asset pair has a direct substitution effect and an indirect complementary effect due to the third asset (for a detailed theoretical analysis, see Driskill & McCafferty, 1987, and Ogaki, 1990). Whether assets end up as net substitutes or complements depends on the underlying parameters describing their risk characteristics. Applying this discussion to our setting of short- and long-term domestic bonds and the foreign exchange rate, it is first noted that the short-term domestic bond has minimal risk for practical purposes. Second, the risk associated with the short-term foreign bond can be proxied by the volatility of the exchange rate return conditional on time. Finally, the risk proxy for the long-term bond is the conditional volatility of long-term rates. The conditional risks of the latter two assets relative to each other lead to four cases.

In one extreme case (which can be labeled as Case 4), the conditional volatility of the long-term domestic bond yield would be higher than that of the exchange rate return. This would imply that the foreign bond acts as a substitute for both short-term and long-term domestic bonds. Moreover, the minimal risk short-term domestic bond and the highly volatile long-term bond would end up as complements of each other because of their very different risk characteristics. The next two cases (Case 3 and Case 2) would correspond to the long-term bond and the foreign bond having similar conditional volatilities; the first slightly higher than the second in Case 3, and vice versa in Case 2. In both cases, the three assets would end up as substitutes of each other since there would be no extreme difference in the risk characteristics of these assets.

The empirical study here is based on the more realistic and plausible Case 1, where the volatility of the exchange rate return is higher than the volatility of long-term interest rates. The fact that the exchange rate return has higher volatility has been empirically justified in Backus et al. (2001).
Moreover, as explained in Section 4 and shown in Panel C of Table 1 below, foreign currency return volatility is much higher than long-term interest rate volatility for every currency that is examined. Long-term interest rate volatility in turn is higher than short-term volatility, demonstrating the validity of Case 1. The long-term bond is more risky than the short-term bond but less risky than the foreign bond and ends up as a substitute for both assets. And the foreign and short-term bonds are complements of each other. The empirical details are discussed in Section 4.

In this theoretical setting, suppose that the domestic short-term interest rate rises. The price of the bond is lower (since yields and prices move in opposite directions); and the investor will want to increase the proportion of this higher return asset while keeping the risk level the same. This will have two opposite effects on the demand for foreign bonds in the portfolio (and thus on the dynamics of the exchange rate). The first, negligible effect is due to the very weak substitution between the short-term bond and the foreign bond. The second is the dominant complementary effect which increases the demand for foreign bonds. This will immediately increase the foreign currency price (domestic currency will depreciate immediately). Empirical investigation of this conjecture should lead to a positive relationship between short-term interest rate differentials and contemporaneous exchange rates.

### Table 1

**Country risk rankings**

<table>
<thead>
<tr>
<th>Panel A. Institutional investor country credit survey</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>GER</td>
</tr>
<tr>
<td>Mean</td>
<td>2.98</td>
</tr>
<tr>
<td>S.E.</td>
<td>1.60</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. Euromoney country risk rankings</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>GER</td>
</tr>
<tr>
<td>Mean</td>
<td>3.79</td>
</tr>
<tr>
<td>S.E.</td>
<td>2.75</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel C. Coefficient of variation (CV) and S.D. of US returns vs. foreign currency returns</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>GER</td>
</tr>
<tr>
<td>S.D.</td>
<td>0.0032</td>
</tr>
<tr>
<td>CV</td>
<td>2.14</td>
</tr>
</tbody>
</table>

Panel A and Panel B present mean and standard deviation of country risk rankings. The lower the mean value of the rank, the lower the country risk for investment purposes. Panel C presents standard deviation and coefficient of variation (CV) of Rs (US short-term rate), Rl (US long-term rate), and seven different currency returns. SWI: Switzerland, GER: Germany, FRA: France, ITA: Italy, JPA: Japan, UK: United Kingdom, BEL: Belgium. The rankings have been compiled and computed from semiannual issues of Euromoney and Institutional Investor publications.

Moreover, as explained in Section 4 and shown in Panel C of Table 1 below, foreign currency return volatility is much higher than long-term interest rate volatility for every currency that is examined. Long-term interest rate volatility in turn is higher than short-term volatility, demonstrating the validity of Case 1. The long-term bond is more risky than the short-term bond but less risky than the foreign bond and ends up as a substitute for both assets. And the foreign and short-term bonds are complements of each other. The empirical details are discussed in Section 4.

In this theoretical setting, suppose that the domestic short-term interest rate rises. The price of the bond is lower (since yields and prices move in opposite directions); and the investor will want to increase the proportion of this higher return asset while keeping the risk level the same. This will have two opposite effects on the demand for foreign bonds in the portfolio (and thus on the dynamics of the exchange rate). The first, negligible effect is due to the very weak substitution between the short-term bond and the foreign bond. The second is the dominant complementary effect which increases the demand for foreign bonds. This will immediately increase the foreign currency price (domestic currency will depreciate immediately). Empirical investigation of this conjecture should lead to a positive relationship between short-term interest rate differentials and contemporaneous exchange rates.

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3 As the demand for the short bond increases, the demand for the long bond decreases, which implies that the demand for foreign currency increases. Short bonds and foreign currency are complements since their demand functions move in the same direction.

4 The strong substitutability between the domestic short-term and long-term bonds means that the demand for long-term bonds decreases. This decrease has the indirect effect of increasing the demand for foreign bonds since long-term and foreign bonds are also strong substitutes.

5 In the next period under rational equilibrium, foreign currency will lose value, i.e., domestic currency will appreciate, which is the forward premium anomaly. In a regression trying to explain the future changes in foreign currency rates, the slope coefficient should be negative for short-term rates consistent with this anomaly.
The relationship between long-term interest rate differentials and exchange rates is more direct. Both long-term bonds and foreign bonds are strong substitutes of each other. Therefore, as the demand for long-term bond decreases, the demand for foreign bond increases leading to an immediate increase in the price of the foreign currency. The empirical consequence of this conjecture is a negative relationship between long-term interest rate differentials and contemporaneous exchange rates. The following section formally introduces the empirical model.

3. Design of empirical tests

Let $F(t, T)$ denote the futures/forward price (in dollars) at time $t$ of a foreign currency with maturity date, $T$. For any $h$ with $0 < h \leq T - t$, the following pricing relation holds under the risk-neutral measure $N$,\(^6\)

$$F(t, T) = E_t^N F(t + h, T).$$

Let $S(t)$ denote the spot price (exchange rate) of the currency at time $t$. When $t + h = T$,

$$F(t, T) = E_t^N F(T, T) = E_t^N S(T).$$

That is, under the risk-neutral measure, the futures price is an unbiased estimate of the future spot exchange rate at $T$. Subtracting $S(t)$ from both sides of Eq. (2) and then dividing the equation by $S(t)$ gives the futures-spot spread as just the expected spot currency return,

$$\frac{F(t, T) - S(t)}{S(t)} = E_t^N \frac{S(T) - S(t)}{S(t)}.$$ \(3\)

In a risk-neutral world, the futures-spot spread (divided by the spot exchange rate) is the best predictor of the future spot currency return.

The general version of the uncovered interest rate parity imposes a constant currency risk premium. This leads to the following regression,

$$\frac{S(T) - S(t)}{S(t)} = \alpha + \beta \frac{F(t, T) - S(t)}{S(t)},$$

under the null hypothesis of $\beta = 1$. The tests on the UIP in the literature have often used log returns rather than percentage returns:

$$\ln S(T) - \ln S(t) = \alpha + \beta [\ln F(t, T) - \ln S(t)].$$ \(5\)

This is qualitatively the same as (4). Most studies then use interest rate differentials to test the UIP\(^7\): \(1 / \tau) [\ln S(T) - \ln S(t)] = \alpha + \beta [Y_d(t, \tau) - Y_f(t, \tau)],\) where $\tau = T - t$ and $Y_d(t, \tau)$ and $Y_f(t, \tau)$ are domestic and

\(^6\) The probability measure the representative agent/investor uses to calculate expectations in a risk-neutral world is called the risk-neutral measure, $N$. In such a risk-neutral world, the representative agent is indifferent to risk and requires no additional compensation for risk. Thus, the level of risk is irrelevant, meaning that there is no penalization for risk. Every investor in this world believes, by definition, that the subjective expected rate of return on each asset is the appropriate discount rate. This theoretical condition is relaxed to a certain extent in empirical analyses. The non-zero intercept coefficient in regression tests is often interpreted as a constant risk premium. For a more detailed discussion, see Backus et al. (2001), and for an introduction Hull (2002, pp. 55 and 244).

\(^7\) See for example, Ahn (2004), Backus et al. (2001), Bansal (1997), Fama (1984), and Inci and Lu (2004).
foreign $\tau$-period nominal interest rates. Since covered interest rate parity, $(1/\tau)[\ln F(t,T) - \ln S(t)] = Y_d(t,\tau) - Y_f(t,\tau)$, always holds by no arbitrage, this is the same as (5). From (5) and covered interest parity, uncovered interest rate parity is tested in the literature mainly with the following regression

$$s(1 + \tau) - s(t) = \beta_0 + \beta_1 (Y_d(t, \tau) - Y_f(t, \tau)) + \epsilon_t,$$

where $s(t + \tau) - s(t)$ is the log return of the foreign currency and $Y_d(t, \tau) - Y_f(t, \tau)$ is the differential of the domestic and foreign interest rates.

One shortcoming of the above model is the forecast horizon (generally picked as 1 month in previous studies). To account for quick reactions to changes in interest rates in today’s professional investment markets, a contemporaneous relationship between exchange rates and interest rates is developed below for the empirical investigation. It is based on Baxter (1994), which uses a sticky price exchange rate model with co-integrating regressions (the origins of the model go to Dornbusch, 1976, and Frankel, 1979). The model assumes that prices of goods adjust slowly to disturbances and come back to their long-run equilibrium value at a constant rate, $\theta$.\(^8\) The log of the real exchange rate is defined as

$$q(t) = s(t) - p_d(t) + p_f(t),$$

where $s(t)$ is the log of the nominal exchange rate, $p_d(t)$ is the log of the domestic consumer price index, and $p_f(t)$ is the log of the foreign consumer price index. The sticky price of the exchange rate is:

$$E_t(q(t + \tau) - \bar{q}(t + \tau)) = \theta^\tau (q(t) - \bar{q}(t)), \quad 0 < \theta < 1,$$

where $E_t$ is the conditional expectation based on all available information at time $t$, and $\bar{q}(t)$ is the real exchange rate that would prevail at time $t$ if all prices are fully flexible. Thus, using $E_t(\bar{q}(t + \pi)) = \bar{q}(t)$, covered interest parity, and Eq. (5), we have

$$q(t) = \rho (y_d(t, \tau) - y_f(t, \tau)) + \bar{q}(t),$$

where $\rho = 1/(\theta^\tau - 1) < 0$, and $y_d(t, \tau)$ and $y_f(t, \tau)$ are domestic and foreign $\tau$-period real interest rates (details of the derivation are in Appendix A). Eq. (8) shows a direct relationship between the contemporaneous real exchange rate and the real interest rates for empirical analysis. If the expectations hypothesis holds, an increase in the domestic real interest rate relative to the foreign rate immediately decreases the real exchange rate. But if the theoretical conjecture of Section 2 is valid, we should see the opposite for short rates: as the domestic short rate increases relative to the foreign short rate, the current exchange rate increases. On the other hand, there should be a negative contemporaneous relationship between long-term interest differentials and foreign currency.\(^9\)

**Edison and Pauls (1993)** have suggested that the relationship between exchange rates and interest rates can be established properly with additional variables. However, one has to be careful in the inclusion of explanatory variables. Unless there is a sound theoretical basis, the arbitrary inclusion of

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\(^8\) $\theta$, always positive and less than 1, is called the coefficient of adjustment. In the limit, if $\theta = 0$, prices are fully flexible, there is no price stickiness, and we end up with the model described in Eq. (6). If $\theta = 1$, prices are expected to stay at their current level regardless of shocks, and never move to their long-run equilibrium.

\(^9\) In the steady state equilibrium next period, the currency will depreciate as suggested by the forward premium puzzle producing a negative slope coefficient between the currency change and short-term interest rate differentials. On the other hand, the long-term interest rate differentials will have positive slope coefficients when regressed against the exchange rate change.
explanatory variables may not provide useful conclusions and tests might be contaminated with over-parameterization. Here the additional explanatory variable is long-term interest rate differentials consistent with the theoretical layout. Therefore, co-integration regressions are performed on the following model:

\[ q(t) = \rho_0 + \rho_1(y_{s,d}(t) - y_{l,f}(t)) + \rho_2(y_{l,d}(t) - y_{l,f}(t)) + \rho_3 t + \varepsilon_t. \] (9)

The sign of \( \rho_2 \) is expected to be negative. The sign of \( \rho_1 \) will be positive if short rates and foreign currency are complements. This will be consistent with the forward premium puzzle. If \( \rho_1 \) ends up as negative, then uncovered interest parity will hold for short- as well as long-term rates.

4. Data and descriptive statistics

Spot currency and government bond yield data are obtained from Data Resources, Incorporated (DRI). The sample extends from June 1973 through April 1998 covering the period of free floating exchange rate systems adopted by the majority of developed economies. Exchange rates are the monthly average of daily rates and are quoted in dollars per unit of foreign currency. For short-term rates, 3-month government bond yields for the United Kingdom, Switzerland, Japan, Italy and Belgium, 3-month Interbank Offered rates for Germany and France (FIBOR and PIBOR), and 3-month Treasury bill rates for the US are used. For long-term interest rates, yields on 10-year government bonds are used. Price changes are measured by the seasonally unadjusted consumer price index data.

The ex-ante real short-term interest rate and the real long-term interest rate are constructed using a measure of expected inflation. To this end, one-period-ahead expected inflation is computed using the 3-month realized inflation rate under the assumption that it follows an autoregressive process. The annualized 3-month expected inflation rate is used as a proxy for short-term expected inflation.\(^{12}\)

The first goal is to determine whether the data are empirically consistent with the theoretical setting presented in Section 2. Is the risk of the short-term domestic bond (represented by the standard deviation of the domestic yield) less than that of the long-term domestic bond? Is the risk associated with the domestic long-term bond less than the risk associated with the foreign bond (measured by the standard deviation of the change in the exchange rate)? Affirmative answers to both questions will correspond to Case 1 described in General theory above.

Using Euromoney rankings (based on spreads of Euromarket borrowings of countries) from March 1985 through 1998 and Institutional Investor rankings (based on surveys of lending officers of largest banks) from March 1979 through 1998, time series of country risk rankings are created for each country. These rankings are provided semiannually in March and September issues of two different publications:

\[^{10}\] Euro notes and coins began circulating in 2002; however most European currencies were placed in strict bands from January 1999. The sample ends around mid-1998 to exclude these artificial restrictions.

\[^{11}\] Interbank Offered rate is subject to slightly more risk because it is a business rate. But this choice does not bias the results in favor of the model; in fact the opposite is true: selecting government rates would support the theory more strongly. Empirical results with Interbank Offered rates are still consistent with theory.

\[^{12}\] The order of the AR process is determined from Schwarz, Akaike, Final Prediction Error, Shibata, and Hannan–Quinn information criteria. In each country, the order suggested by most criteria is used. It is hard to obtain accurate expected inflation over a 10-year horizon. Following Baxter (1994), 3-month expected inflation is used for both short and long-term expected inflation.
Euromoney and Institutional Investor. The last set of rankings used in this article can be found at Euromoney Publications (1999) and Institutional Investor (1999). Moments of the rankings are presented in Table 1. The results indicate the US as the least risky country.13 Thus the domestic country, the US, is regarded as the least risky country in the two-country empirical analysis in this study. More specifically, a long-term US bond can be conjectured to have less risk than a foreign bond. However this particular conjecture must be justified empirically. This is achieved with the descriptive statistics on the data. The last panel of the table presents the standard deviation and coefficient of variation (CV) values as alternative measures of risk for the domestic short rate, the domestic long rate, and the foreign currency. Every currency exhibits much higher risk than those of the domestic rates for either measure. Additionally, consistent with theory, long-term rates exhibit more risk than short rates. The CV of each currency is at least 5 times more than the long-term CV of 4.78, while the CV of short rate is 2.14. One can conclude from these cross-sectional results that long-term bonds are more risky than short-term bonds, while foreign currencies have the highest risk level.

Table 2 is about the variables used in the regressions. Panel A presents the means and standard deviations of interest rate differentials between the US and each foreign country. Sample moments of

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Table 2
Properties of interest rates and currencies

Panel A: Summary statistics: interest rates

<table>
<thead>
<tr>
<th>Currency</th>
<th>( y_{s,d} - y_{s,f} ) Mean</th>
<th>S.D.</th>
<th>( y_{l,d} - y_{l,f} ) Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>German mark</td>
<td>-0.0013</td>
<td>0.0030</td>
<td>-0.0029</td>
<td>0.0039</td>
</tr>
<tr>
<td>Swiss franc</td>
<td>0.0001</td>
<td>0.0036</td>
<td>-0.0007</td>
<td>0.0040</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>-0.0003</td>
<td>0.0034</td>
<td>-0.0018</td>
<td>0.0051</td>
</tr>
<tr>
<td>French franc</td>
<td>-0.0012</td>
<td>0.0030</td>
<td>-0.0027</td>
<td>0.0035</td>
</tr>
<tr>
<td>British pound</td>
<td>-0.0016</td>
<td>0.0038</td>
<td>-0.0025</td>
<td>0.0047</td>
</tr>
<tr>
<td>Belgian franc</td>
<td>-0.0019</td>
<td>0.0033</td>
<td>-0.0030</td>
<td>0.0042</td>
</tr>
<tr>
<td>Italian lira</td>
<td>-0.0018</td>
<td>0.0037</td>
<td>-0.0029</td>
<td>0.0038</td>
</tr>
</tbody>
</table>

Panel B: Summary statistics: currency prices

<table>
<thead>
<tr>
<th>Currency</th>
<th>( Q_t ) Mean</th>
<th>S.D.</th>
<th>Depreciation rate, ( Q_{t+1}/Q_t - 1 ) Mean</th>
<th>S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>German mark</td>
<td>0.5892</td>
<td>0.0903</td>
<td>0.0003</td>
<td>0.0330</td>
</tr>
<tr>
<td>Swiss franc</td>
<td>0.6489</td>
<td>0.1086</td>
<td>0.0015</td>
<td>0.0373</td>
</tr>
<tr>
<td>Japanese yen</td>
<td>0.0070</td>
<td>0.0016</td>
<td>0.0009</td>
<td>0.0345</td>
</tr>
<tr>
<td>French franc</td>
<td>0.1690</td>
<td>0.0243</td>
<td>0.0006</td>
<td>0.0319</td>
</tr>
<tr>
<td>British pound</td>
<td>1.5816</td>
<td>0.2115</td>
<td>0.0010</td>
<td>0.0347</td>
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<tr>
<td>Belgian franc</td>
<td>0.0286</td>
<td>0.0051</td>
<td>0.0004</td>
<td>0.0336</td>
</tr>
<tr>
<td>Italian lira</td>
<td>0.0007</td>
<td>0.0001</td>
<td>-0.0001</td>
<td>0.0318</td>
</tr>
</tbody>
</table>

Sample period is from June 1973 through April 1998. Panel A presents mean and standard deviation short-term differential between the domestic \( y_{s,d} \) and foreign \( y_{s,f} \) yields followed by long-term differential. Panel B is mean and standard deviation of real currency prices (\( Q_t \)), and changes in real exchange rate.

---

13 Statistically there is no difference between Switzerland and the US. The null hypothesis of difference of the means of the US and Swiss rankings being equal to zero cannot be rejected at 5% significance. Considering only Euromoney rankings increases the significance even more.
short-term interest rate differentials are followed by those of long-term interest rate differentials. Panel B presents the sample moments for real currencies in levels and returns.

The cross-sectional results show that the currency data have much higher volatility than the corresponding interest rate differentials. Standard deviations of the currencies are higher than those of the short and long-term interest rate differentials by as much as 10 times. In addition, long-term differentials have higher standard deviations compared to short-term deviations.14

The results support Case 1 described in General theory above and are also consistent with previous studies such as those of Backus et al. (2001) and Baxter (1994). Foreign currency/bond has much higher risk than short-term domestic bond making it a substitute for the long-term domestic bond only. The domestic long-term bond, on the other hand, can also be a rough substitute for the short-term bond since it is less risky than the foreign currency. Foreign bond acts therefore as substitute of a substitute for the short-term bond. They are at opposite ends of the risk spectrum and are complements of each other.

Next, immediate and steady state reactions of the currency to short-term rate changes are examined. The goal is to answer the question: is there indeed a positive contemporaneous relationship between the currency and the short rate difference demonstrating the forward premium puzzle?

5. Empirical results

5.1. Unit root tests

Previous studies report that real currencies or interest differentials are integrated of order one, I(1), meaning that the data is nonstationary, or has unit roots (Meese & Rogoff, 1988). Such data are also said to have long memory, where an old shock will continue having an impact on current values. Co-integrating regressions should be conducted in such cases.

To determine whether the data contain unit roots, five different tests are conducted. Three of them are independent versions of augmented Dickey and Fuller tests (Fuller, 1996): (1) ADF is based on estimations of the OLS $t$-statistic; (2) ADF2 is the rho test based on estimations of the OLS autoregressive coefficients; and (3) ADF3 is based on the OLS F statistics.15 Serial correlations in the tests are controlled by including higher-order autoregressive terms following Said and Dickey (1984). Since the power of a unit root test is low in general, two additional tests are used: Phillips–Perron $Z_q$ unit root test and Phillips–Perron $Z_t$ unit root test (Perron, 1988).16 Results are reported in Table 3. Overall, neither the Dickey–Fuller nor the Phillips–Perron tests reject the null hypothesis of difference

---

14 The statistics on foreign currencies also demonstrate the difficulty researchers have in trying to forecast exchange rate returns as explained in detail by Engel and West (2004).

15 The model used in each augmented Dickey–Fuller test is $z(t) = \xi_1 \Delta z(t-1) + \cdots + \xi_{p-1} \Delta z(t-p+1) + \eta z(t-1) + \varepsilon(t)$, where $z$ is the time series of interest with $T$ observations. In ADF, the null hypothesis is $\eta = 1$, i.e., there is a unit root, and the test statistic is $(\hat{\eta} - 1)/(\text{standard error of } \hat{\eta})$. Critical values for the distribution are provided in Fuller (1996, p. 642). In ADF2, the null is $\eta = 1$ and data follows an autoregressive process. The test statistic is $T(\hat{\eta} - 1)/(1 - \hat{\xi}_1 - \cdots - \hat{\xi}_{p-1})$, and has a distribution described in Fuller (1996, p. 641). In ADF3, the null is $\eta = 1$ and $\gamma = 0$. The OLS F test statistics for the hypothesis involving two restrictions are described in Fuller (1996) and provided in Dickey and Fuller (1981, p. 1063).

16 The model in both Phillips–Perron tests is $z(t) = \eta_1 z(t-1) + \varepsilon(t)$, and the null hypothesis is $\eta = 1$. The test statistics are provided in detail in Perron (1988) and the critical distribution values are in Fuller (1996), page 641 for Phillips–Perron $Z_p$ test, and page 642 for Phillips–Perron $Z_t$ test.
stationarity for any of the data at 1% significance. The only two exceptions are the augmented Dickey–Fuller rho test of the long-term real interest rate differential for Switzerland, and the short-term real interest rate differential for Belgium. Since all data are nonstationary at least by one test and since results indicate that real long- and short-term interest differentials possess a unit root or at least near unit root,

### Table 3
Unit root tests

<table>
<thead>
<tr>
<th>US vs.</th>
<th>Unit root test</th>
<th>$q_t$</th>
<th>$y_{s,d} - y_{s,f}$</th>
<th>$y_{l,d} - y_{l,f}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>ADF</td>
<td>-1.5586</td>
<td>-2.2908</td>
<td>-2.8133</td>
</tr>
<tr>
<td></td>
<td>ADF2</td>
<td>-4.7680</td>
<td>-16.7711</td>
<td>-15.4989</td>
</tr>
<tr>
<td></td>
<td>ADF3</td>
<td>0.0039908</td>
<td>0.095144</td>
<td>1.9511</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_p$</td>
<td>-5.3546</td>
<td>-6.3615</td>
<td>-9.6298</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_t$</td>
<td>-1.6490</td>
<td>-1.7895</td>
<td>-2.2349</td>
</tr>
<tr>
<td>Switzerland</td>
<td>ADF</td>
<td>-1.7762</td>
<td>-1.5022</td>
<td>-3.3290</td>
</tr>
<tr>
<td></td>
<td>ADF2</td>
<td>-5.9890</td>
<td>-6.1869</td>
<td>-21.3112</td>
</tr>
<tr>
<td></td>
<td>ADF3</td>
<td>0.0061737</td>
<td>0.34657</td>
<td>6.5113</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_p$</td>
<td>-6.6421</td>
<td>-6.2457</td>
<td>-14.3864</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_t$</td>
<td>-1.8659</td>
<td>-1.7855</td>
<td>-2.7532</td>
</tr>
<tr>
<td>Japan</td>
<td>ADF</td>
<td>-1.9738</td>
<td>-1.3181</td>
<td>-1.9669</td>
</tr>
<tr>
<td></td>
<td>ADF2</td>
<td>-3.9220</td>
<td>-3.9319</td>
<td>-5.5652</td>
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<tr>
<td></td>
<td>ADF3</td>
<td>1.2548</td>
<td>0.045505</td>
<td>1.5426</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_p$</td>
<td>-4.5585</td>
<td>-2.9523</td>
<td>-5.1072</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_t$</td>
<td>-2.0784</td>
<td>-1.0710</td>
<td>-2.0593</td>
</tr>
<tr>
<td>France</td>
<td>ADF</td>
<td>-1.5682</td>
<td>-2.0367</td>
<td>-1.3379</td>
</tr>
<tr>
<td></td>
<td>ADF2</td>
<td>-4.9828</td>
<td>-10.4475</td>
<td>-4.1527</td>
</tr>
<tr>
<td></td>
<td>ADF3</td>
<td>0.048746</td>
<td>1.1847</td>
<td>1.8637</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_t$</td>
<td>-1.5954</td>
<td>-2.5035</td>
<td>-1.3855</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>ADF</td>
<td>-2.0423</td>
<td>-2.3218</td>
<td>-2.2738</td>
</tr>
<tr>
<td></td>
<td>ADF2</td>
<td>-8.0231</td>
<td>-14.3526</td>
<td>-10.6504</td>
</tr>
<tr>
<td></td>
<td>ADF3</td>
<td>0.015628</td>
<td>7.3023</td>
<td>17.054</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_t$</td>
<td>-2.1549</td>
<td>-2.6425</td>
<td>-1.7693</td>
</tr>
<tr>
<td>Belgium</td>
<td>ADF</td>
<td>-1.4438</td>
<td>-3.4460</td>
<td>-2.0653</td>
</tr>
<tr>
<td></td>
<td>ADF2</td>
<td>-3.9339</td>
<td>-24.9953</td>
<td>-8.7304</td>
</tr>
<tr>
<td></td>
<td>ADF3</td>
<td>0.24872</td>
<td>2.7390</td>
<td>4.8683</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_p$</td>
<td>-4.2647</td>
<td>-13.6436</td>
<td>-5.2517</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_t$</td>
<td>-1.5027</td>
<td>-2.5589</td>
<td>-1.5904</td>
</tr>
<tr>
<td>Italy</td>
<td>ADF</td>
<td>-1.4002</td>
<td>-1.7152</td>
<td>-1.5541</td>
</tr>
<tr>
<td></td>
<td>ADF2</td>
<td>-3.5784</td>
<td>-7.7648</td>
<td>-3.2362</td>
</tr>
<tr>
<td></td>
<td>ADF3</td>
<td>7.0684</td>
<td>19.641</td>
<td>0.11776</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_p$</td>
<td>-4.2757</td>
<td>-5.9455</td>
<td>-3.2992</td>
</tr>
<tr>
<td></td>
<td>Phillips–Perron $Z_t$</td>
<td>-1.5437</td>
<td>-1.4727</td>
<td>-1.5727</td>
</tr>
</tbody>
</table>

$q_t$: real currency; $y_{s,d} - y_{s,f}$ short-term real interest rate difference; $y_{l,d} - y_{l,f}$ long-term real interest rate difference. If ADF>critical value, accept the null of a unit root. If ADF2>critical value, accept the null of a unit root and that the data follows an AR process with no constant term. If ADF3<critical value, accept the null of a unit root and no constant term. Phillips–Perron $Z_p$ and Phillips–Perron $Z_t$ are the first two tests on page 308 of Perron (1988). For both tests accept the null that there is a unit root if the test statistic is higher than the critical value. For ADF and Phillips–Perron $Z_t$, critical values are $-3.485, -2.885, -2.575$; for ADF2 and Phillips–Perron $Z_p$, the critical values are $-20.05, -13.85, -11.1$; and for ADF3 the critical values are 6.61, 4.66, and 3.835 at 1%, 5%, and 10%, respectively.
the relationship between real exchange rates and real long- and short-term interest differentials should be determined using co-integrating regressions.

Before proceeding with the Phillips–Hansen fully modified OLS co-integrating regression, the explanatory variables need to be investigated to determine whether they are themselves co-integrated with one another (Phillips & Hansen, 1990). If they are, then there is more than one co-integrating relation between the exchange rate, real short-, and real long-term interest rate variables. Table 4 reports the results of augmented Dickey–Fuller, Phillips’ $Z_q$ and $Z_t$ co-integration tests between real long- and real short-term interest rate differentials. Results fail to reject the null of no co-integration for any country at a significance level of 10%.

5.2. Co-integrating regressions

The last result of the previous section allows proceeding with Phillips and Hansen’s fully modified OLS co-integrating regression. Real exchange rates are regressed on short- and long-term real interest rate differentials along with a constant term and a deterministic trend term. The results are reported in Panel A of Table 5. Point estimates have signs consistent with the conjectures presented in Section 2 for every country. The point estimates for real long-term interest rates range from $-4$ to $-1.5$ and are significant in all cases. Higher long-term real interest rate in the US relative to foreign long-term real interest rate causes real and immediate appreciation of the dollar. In long-run equilibrium, the dollar depreciates as suggested by the expectations hypothesis. Therefore, the UIP holds for long horizon rates. These results are consistent with those reported by Alexius (1993) and Meredith and Chinn (1998).

The slope coefficients for short-term real interest rate differentials are all positive and significant; and are greater than 1.5 for most countries. Positive point estimates indicate that foreign and domestic short-term bonds are complements. This is consistent with Case 1 of the theoretical discussion in Section 2. For the UK, the short-term coefficient has the smallest point estimate. Although statistically significant, the small value may hint that the US–UK dynamics is shifting to Case 2 or Case 3, where all assets are alternatives of one another. In other words, foreign bonds are not much more risky than long-term bonds. This would not be too surprising. United Kingdom is the largest foreign investor in the United States. Foreign investment by Britain has been growing at the fastest rate among the countries investigated here (InterSec Research Corporation, 2001). These might be reasons why, for US investors, investing in the British pound denominated assets may not be considered as risky investments in comparison to those denominated in other currencies.

Table 4
Co-integration among explanatory variables

<table>
<thead>
<tr>
<th>US vs.</th>
<th>Germany</th>
<th>Switzerland</th>
<th>Japan</th>
<th>France</th>
<th>UK</th>
<th>Belgium</th>
<th>Italy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co-integration tests $Z_q$</td>
<td>$-14.2400$</td>
<td>$-1.0842$</td>
<td>$-0.1308$</td>
<td>$-0.1276$</td>
<td>$-0.6857$</td>
<td>$-0.3931$</td>
<td>$-0.0486$</td>
</tr>
<tr>
<td>$Z_t$</td>
<td>$-0.7312$</td>
<td>$-0.3299$</td>
<td>$-0.8380$</td>
<td>$-0.7300$</td>
<td>$-0.1697$</td>
<td>$-0.0540$</td>
<td>$-0.1475$</td>
</tr>
<tr>
<td>ADFt</td>
<td>$-0.8077$</td>
<td>$-0.3486$</td>
<td>$-0.6335$</td>
<td>$-0.8972$</td>
<td>$-0.3273$</td>
<td>$-0.2548$</td>
<td>$-0.0149$</td>
</tr>
</tbody>
</table>

$Z_q$: Phillips $Z_q$ statistic. If $Z_q < \text{critical value}$, there is evidence against no co-integration. Critical values are $-28.3$, $-20.5$, and $-17.0$ at 1%, 5%, and 10%. $Z_t$: Phillips $Z_t$ statistic. If $Z_t < \text{critical value}$, there is evidence against no co-integration. ADFt: Augmented Dickey–Fuller t Statistic. If ADFt $< \text{critical value}$, there is evidence against no co-integration. Critical values for both $Z_t$ and ADFt tests are $-3.39$, $-2.76$, and $-2.45$ at 1%, 5%, and 10%.

17 Refer to Phillips and Hansen (1990) for technical details of the co-integrating regression procedure.
Although not reported in Table 5, the intercept, $q_0$, is nonzero for the majority of regressions (which is statistically significant only for some currencies). It reflects a constant exchange rate risk premium component demanded by investors on foreign vs. domestic assets.

### 5.3. Robustness tests

#### 5.3.1. Johansen’s fully specified error correction model

Although the Phillips–Hansen regression has been used in various studies, Johansen’s maximum likelihood error correction model is also very popular (Johansen, 1988). In fact, Gonzalo (1994) has shown that it has better properties than the OLS, nonlinear least squares, principle component, and canonical correlation estimators. Therefore, as a robustness check, the empirical analysis is replicated with Johansen’s model. The results are reported in Table 5, Panel B. Neither trace nor eigenvalue test statistics reject the null of no co-integration. The normalized eigenvector coefficients are consistent with previous regressions and with the theoretical model described in Section 2: short rate differences have positive (larger than 1) and long-term differences have negative (less than $-1$) coefficients in explaining the currency. Critical values for trace and eigenvalue statistics are 35.46 and 25.87 at 1%. \[18\]

Although not reported in Table 5, the intercept, $q_0$, is nonzero for the majority of regressions (which is statistically significant only for some currencies). It reflects a constant exchange rate risk premium component demanded by investors on foreign vs. domestic assets.

#### 5.3.2. Currency depreciation vs. interest rate differentials

In standard studies of currency–interest rate relationships, exchange rate depreciation is used as the dependent variable as in Eq. (6). In this section, the same procedure is followed to determine whether the conclusions of the previous sections are robust to this new set of regressions. The log difference of the

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18 Johansen’s procedure is known to be sensitive to the number of lag terms. Therefore regressions with up to 10 lags were conducted. Results and conclusions continue to hold for these regressions.
real exchange rate is regressed on short-term and long-term domestic and foreign real interest rate differentials. The exchange rate change will have at most a fractal root after this difference. But the existence of unit roots in explanatory variables makes the cointegration procedure also appropriate for the tests. First, OLS regressions are reported in Panel A of Table 6. As anticipated, the short-term rate differential has a negative slope coefficient which is significantly less than one for every currency, consistent with the forward premium puzzle. (The only exception is Italy where the coefficient is positive but still significantly less than one.) On the other hand, the long-term rate differential has a positive and significant slope coefficient consistent with the expectations hypothesis and uncovered interest parity. Exact conclusions are obtained from Panel B of Table 6 where co-integrating regressions are employed. Alexius (1993) and Meredith and Chinn (1998) come up with similar results using different techniques in their investigation of long-horizon interest rates and currencies.

6. Conclusion

This paper proposes a relationship between currencies and short- and long-term interest rates. Exchange rates depend on both short-term and long-term rate differentials. As central banks change short-term rates to affect the currency, the reaction of long-term rates to these changes should also be taken into account because of the indirect influence of long-term rates on currencies. The term structure as a whole will determine the final impact of monetary policies on currency movements.

Co-integrating regressions are performed on contemporaneous currency rates with real short-term and real long-term interest rate differentials as explanatory variables. Regressions generate negative and significant coefficients for long-term rate differentials. This is consistent with uncovered interest parity. Therefore, the expectations hypothesis seems to be valid for long horizons. Positive coefficients for real short-term interest rate differentials are consistent with the forward premium puzzle, which is the failure of uncovered interest parity for short-horizons. The results are robust to a new set of regressions where the dependent variable is the future currency change. These conclusions are consistent with the theoretical conjectures of the paper: domestic short-term bonds and foreign bonds have very different risk characteristics. In a portfolio of securities, they serve as complements of each other. On the other
hand, the long-term domestic bond is a suitable substitute for both the short-term domestic bond and the foreign currency.

The recent introduction of the euro neither changes nor diminishes the results presented in the study. For practical purposes, British pound, Japanese Yen, and Swiss franc are not part of the euro. Systematic analysis of the relationship between the euro and the German mark is warranted for future research when more data becomes available. If a strong link between the euro and the German mark is found (given the economic and financial influence of Germany in the European Union), the results and conclusions of this study will apply to the euro as well.

Acknowledgements

I would like to thank Nejat Seyhun, Gautam Kaul, Josh Coval, Biao Lu, David Humphrey, Jeff Clark, and James Nelson for their suggestions and seminar participants at University of Michigan, Florida State University, and Sabanci University for their comments. I also thank two anonymous referees very much for their invaluable recommendations.

Appendix A

The sticky price version of the monetary exchange rate model assumes that the prices of goods adjust slowly to disturbances and come back to their long-run equilibrium value at a constant rate, $\theta$, which is positive and less than 1. The real exchange rate is defined as

$$Q(t) = S(t) / [P_d(t)/P_f(t)],$$

where $S(t)$ is the exchange rate in dollars per unit of foreign currency, $P_d(t)$ and $P_f(t)$ are the domestic and foreign consumer price index values, respectively. Taking logarithms of both sides gives the equivalent representation as

$$q(t) = s(t) - p_d(t) + p_f(t),$$

where $s(t)$ is the log of the nominal exchange rate, $p_d(t)$ is the log of the domestic consumer price index, and $p_f(t)$ is the log of the foreign consumer price index. The sticky price of the exchange rate states that:

$$E_t(q(t + \tau) - \tilde{q}(t + \tau)) = \theta^\tau (q(t) - \tilde{q}(t)), \quad 0<\theta<1,$$

where $E_t$ is the conditional expectation based on all available information at time $t$, $\theta$ is the coefficient of adjustment (which takes a value between 0 and 1), $\tau$ is the forecast horizon, $\tilde{q}(t)$ is the real exchange rate that would prevail at time $t$ if prices are fully flexible. This means,

$$E_t(\tilde{q}(t + \tau)) = \tilde{q}(t).$$

Substituting (A.4) into (A.3) and rearranging,

$$\theta^\tau q(t) = E_t(q(t + \tau)) - \tilde{q}(t) + \theta^\tau \tilde{q}(t).$$
Finally, subtracting $q(t)$ from both sides and dividing both sides by $\theta^s - 1$, one can find

$$q(t) = \rho (E_t(q(t + \tau)) - q(t)) + \bar{q}(t),$$  \hspace{1cm} (A.6)$$

where $\rho = 1/(\theta^s - 1) < 0$. From uncovered interest parity (which says that the expected currency change is equal to the domestic and foreign interest rate difference in real terms) and (A.6), we have

$$q(t) = \rho (y_d(t, \tau) - y_F(t, \tau)) + \bar{q}(t).$$  \hspace{1cm} (A.7)$$

References


