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International Journal of Industrial Organization
24 (2006) 45–67

International Journal of Industrial Organization

www.elsevier.com/locate/econbase

Multihoming and compatibility ☆

Toker Doganoglu [a,1], Julian Wright [b,*]

[a] Center for Information and Network Economics, University of Munich, Germany
[b] Department of Economics, National University of Singapore, Singapore

Received 26 February 2004; received in revised form 3 November 2004; accepted 17 July 2005
Available online 23 September 2005

Abstract

When competing firms make their services compatible, consumers enjoy greater network benefits. These benefits can also be realized if firms remain incompatible and consumers multihome by purchasing from each of the firms. We find that such multihoming may be a poor substitute for compatibility. Multihoming weakens competition and introduces costs that firms do not internalize. As a result, multihoming can increase the social desirability of compatibility, while making compatibility less attractive for firms. Surprisingly, policymakers should generally be more concerned about the lack of compatibility when multihoming is present. Our results extend to two-sided markets.
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JEL classification: D43; L15; L20; L50

Keywords: Multihoming; Compatibility; Network effects; Standards; Two-sided markets

1. Introduction

In an increasing number of situations, agents purchase two competing products in order to reap maximal network benefits. Consumers may purchase both Microsoft Windows and

☆ We would like to thank two anonymous referees and our editor, Simon Anderson, for their very helpful comments.

* Corresponding author.
E-mail address: jwright@nus.edu.sg (J. Wright).

1 I would like to thank Volkswagen Stiftung for the generous financial support which made this research possible.

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doi:10.1016/j.ijindorg.2005.07.004

Linux, merchants may accept several types of credit cards, people may sign up to AOL and MSN instant messaging services, and game developers may produce games for multiple competing platforms. Such behavior has been termed "multihoming" by Caillaud and Jullien (2003) and Rochet and Tirole (2003).[2]

Multihoming was commonplace during the early introduction of the telephone in the United States. At the start of the 1900s, there were competing but not interconnected telephone networks in many cities. In the absence of interconnection, users would require separate phone lines connecting to each network so as to reach a wider number of people. According to a 1910 Bell survey of Louisville, Kentucky, the rate of multihoming was almost 100% among large-scale enterprises while being under 15% for neighborhood shops and residences (Mueller, 1989, pp. 255–61). In contrast, the re-emergence of competing telecommunications networks in the 1990s was based on mandated interconnection between networks, so that users only required a single phone connection to obtain maximal network benefits.

Despite compatibility being mandated in the telecommunications sector, in other network markets such as payment systems and game platforms there seems to be no serious consideration of network compatibility, arguably because of the presence of widespread multihoming in these markets. For example, in the merger between AOL and Time Warner, the fact that consumers often subscribed to multiple instant messaging services (such as AOL, Microsoft, and Yahoo!) was used as an argument for not requiring compatibility between these networks.[3] This raises the question: does the ability of consumers to multihome mean policymakers need not be concerned about a possible "compatibility problem," in which firms do not make their networks compatible even though doing so is socially desirable?

Our answer is two-fold. The possibility of multihoming makes the compatibility problem more likely to arise, but on the other hand, firms are less likely to inefficiently choose to become compatible. To be clear, we are not advocating that compatibility necessarily be imposed in such settings. Instead, our point is that the presence of widespread multihoming is not a justification for ignoring the issue of compatibility. Multihoming is not always a good substitute for compatibility.

Surprisingly, the existing literature on compatibility and standards remains largely silent on the issue. The literature, starting with Katz and Shapiro (1985) and Farrell and Saloner (1985), almost uniformly assumes that consumers can purchase only one of the competing products (for example, VHS versus BETA) and that there are homogenous network benefits across consumers. An exception is De Palma et al. (1999) who show that double purchases "*drastically affect the nature of the product market equilibrium as well as compatibility choices made by the firms*" (p. 209). Theirs is a model of quantity competition between two firms in which consumers are heterogeneous with respect to network benefits but not with respect to how they value the firms' products. The model

[2] Multihoming was originally an Internet term referred to when a host has more than one connection to the Internet. For instance, multihoming captures the technique of connecting a host to the Internet via two or more Internet Service Providers (ISPs) to maintain network connectivity even if one connection fails. Multihoming has been analyzed in this context by Crémer et al. (2000).

[3] See Section 2.6 for a detailed discussion of this case.

implies, in the absence of multihoming, that firms will always differentiate their product vertically by offering networks of different sizes. The possibility of multihoming eliminates this vertical differentiation and results in a continuum of symmetric equilibria. Despite the complex picture they obtain, they are able to use numerical analysis to conclude that in the presence of multihoming, firms are too likely to become compatible when it is inefficient and that welfare is maximized by imposing complete incompatibility, relying instead on multihoming.[4]

In this paper, we start by considering a simple price-setting model of two differentiated firms that compete in the presence of multihoming. We assume that there are two types of consumers who have different valuations on network benefits, which is required to ensure that multihoming can arise in equilibrium. When the firms' services are not compatible, parameter values are assumed so that high types will choose to multihome while low types will not. For example, in the case of telecommunications, the high types could represent business users while the low types could represent residential users.

In the absence of multihoming, the model is one in which firms have excessive incentives to choose compatibility. Firms will sometimes choose compatibility even though it is not socially desirable, but will never choose incompatibility when it is inefficient. The ability of consumers to multihome affects this divergence between private and social incentives for compatibility in several fundamental ways. Under multihoming, some consumers buy twice, increasing each firm's total sales. This provides firms with an incentive to remain incompatible, even though there is no corresponding social gain of double purchase relative to the alternative of moving to compatibility. The fact that some consumers buy both products also means that consumer expectations are less responsive to price changes. This not only shifts each firm's residual demand upwards but also makes it less price sensitive relative to each firm's residual demand when multihoming is not allowed. These two effects allow firms to sustain higher prices in a multihoming equilibrium. By relaxing price competition, the presence of some multihoming consumers reduces any excessive incentive that firms would otherwise have to become compatible. At the same time, multihoming also results in duplicated costs since consumers buy twice. A benefit of compatibility that firms fail to internalize is the elimination of these duplicated costs. Thus, firms may prefer incompatibility in the presence of multihoming even when compatibility is socially desirable. By the same token, the likelihood of firms inefficiently choosing compatibility is reduced under multihoming.

We generalize our results to a two-sided market setting.[5] Two-sided markets involve two distinct types of users, each of which values the number of users of the other type, and platform(s) that sell to both types of users. As Evans (2002, p. 42) notes "*Most two-sided markets we observe in the real world appear to have several competing two-sided firms and at least one side appears to multihome.*" For instance, some people hold both

[4] An early paper to consider multihoming in a network context is Church and King (1993), who consider the equilibrium versus socially optimal level of learning of a second language.

[5] Armstrong (2005) considers models of two-sided markets in which all agents of one type multihome and all agents of the other type do not. Caillaud and Jullien (2003), and Rochet and Tirole (2003) analyze two-sided market structures allowing agents to multihome. None of these papers considers the implications of multihoming on network compatibility.

MasterCard and Visa credit cards, and most retailers accept both types of cards. Similarly, some people own both PlayStation and Xbox game consoles, and many game developers produce games to run on both platforms. We show that our results are robust to moving to a two-sided market framework. The model in this section also contributes to the growing literature on two-sided markets, by analyzing the realistic case of (partial) multihoming on both sides of the market.

We then consider the analysis in the case without product differentiation. In a one-sided market with strong network effects, it is natural to consider an equilibrium in which all consumers buy from one firm. In this case, allowing consumers to multihome may be an irrelevant option, as consumers already get maximal network benefits buying from the dominant firm. The dominant firm will enjoy high profits and will oppose any move to compatibility. The social planner will share this preference if it is only concerned with total surplus. On the other hand, if the social planner prefers lower prices, it may favor a solution with compatibility but the dominant firm will not sponsor any such move. Then, the ability of consumers to multihome does not solve (or change) any compatibility problem that arises when there are strong network effects.

In a two-sided market setting without product differentiation, the dominant platform will continue to block compatibility. However, we show that the possibility of multihoming may mean that compatibility is now socially desirable even when the social planner is just concerned with total surplus. Multihoming (on one side of the market) can now arise in equilibrium. The social planner will compare the costs of duplicated purchases under this multihoming outcome to the costs of achieving compatibility directly. Where the costs of duplicated purchases are higher, the social planner prefers compatibility even though the platforms do not. A compatibility problem arises due to the ability of agents to multihome.

The rest of the paper proceeds as follows. Section 2 develops a model in which consumers can purchase from one or both of two symmetric firms. The model is used to compare the case with and without multihoming, and with and without compatibility. The model is extended to a two-sided market setting in Section 3, while Section 4 considers a version without product differentiation. Section 5 offers some brief concluding thoughts.

2. Hotelling model of multihoming

We start with a standard Hotelling model of competition with network effects similar to that of Farrell and Saloner (1992). Other similar models include those in Shy (2001), Armstrong (2005), and Griva and Vettas (2004). We extend this standard approach by allowing consumers to be heterogeneous in terms of their marginal valuations of network size so that multihoming arises as an equilibrium outcome.

There are two symmetric firms denoted 1 and 2 which provide a service to consumers at the constant marginal cost, f. Consumers can subscribe to a service from either firm 1, firm 2, or both firms if this is possible (multihoming). Subscribing to a service gives consumers network benefits that are linear in the number of other agents that the consumer can access through the service. There are two types of consumers according to their marginal valuation of the network size, denoted as b. A fraction λ of consumers value the network benefits highly (high types) and have $b=b_{\mathrm{H}}$. The remaining consumers, a fraction of $1-\lambda$, do not value the network benefits highly (low types), and have $b=b_{\mathrm{L}}\geq 0$. Naturally, we assume that $b_{\mathrm{H}}>b_{\mathrm{L}}$ and $0<\lambda<1$.

The net utility of a consumer of type b located at $x\in[0,1]$ when she purchases from firm i is given by

$$U_i(x,b,N_i)=v-p_i-t_i(x)+bN_i, \tag{1}$$

for $i=1$, 2, where v is the intrinsic benefit[6] of the service, p_i is the (uniform) subscription price of firm i, transportation costs $t_i(x)$ equal tx for firm 1 and $t(1-x)$ for firm 2, and N_i represents the total number of consumers that can be reached by subscribing to firm i. When the same consumer multihomes, subscribing to both firms, the net utility she gets is

$$U(x,b,N)=v-p_1-p_2-t_1(x)-t_2(x)+bN,$$

where N represents the total number of consumers that can be reached by subscribing to both firms.[7] The utility of a multihoming customer can be further simplified to

$$U(x,b,N)=v-p_1-p_2-t+b, \tag{2}$$

given $N=1$ (multihoming ensures all consumers can be reached) and $t_1(x)+t_2(x)=t$ (the total distance of travelling to both firms is always unity).

We assume that firms set prices in stage 1 and consumers subscribe to one or both firms in stage 2. We look for subgame perfect equilibria, which implies that consumers form rational expectations to determine the size of each network given the prices set in stage 1.

2.1. Incompatible firms without multihoming

This section provides a benchmark for later results as it corresponds to the existing literature which ignores the possibility of multihoming. The case without multihoming also corresponds to a situation where firms choose to make their services exclusive. Let s_i and n_i denote the share of high and low types that subscribe to firm i respectively. Since there are λ high types and $1-\lambda$ low types, the total number of consumers that can be reached by subscribing to firm 1 is $N_1=\lambda s_1+(1-\lambda)n_1$. Likewise $N_2=\lambda s_2+(1-\lambda)n_2$. The locations of the indifferent consumers at each segment must solve $U_1(s_1, b_{\mathrm{H}}, N_1)=U_2(s_1, b_{\mathrm{H}}, N_2)$ and $U_1(n_1, b_{\mathrm{L}}, N_1)=U_2(n_1, b_{\mathrm{L}}, N_2)$, which, after simplifications, yield

$$s_1=\frac{1}{2}+\frac{p_2-p_1}{2t}+\frac{b_{\mathrm{H}}(p_2-p_1)}{2t(t-\beta)}$$

$$n_1=\frac{1}{2}+\frac{p_2-p_1}{2t}+\frac{b_{\mathrm{L}}(p_2-p_1)}{2t(t-\beta)},$$

[6] We assume that the intrinsic benefit, v, is sufficiently high that all consumers subscribe to at least one firm throughout the paper.

[7] We treat transportation costs literally and sum them. Possible interpretations of this include costs of signing up for a service, or the initial set up costs required for adopting a product or service. However, intrinsic benefits are only obtained once.

where $s_2=1-s_1$, $n_2=1-n_1$ and $\beta=\lambda b_H+(1-\lambda)b_L$ is the average value of the network benefits parameter b. Using these expressions

$$N_1=\frac{1}{2}+\frac{p_2-p_1}{2(t-\beta)}, \tag{3}$$

and given that the market is covered, $N_2=1-N_1$. Note that the market shares are more sensitive to prices than a normal Hotelling model due to the effect of prices on consumer expectations about network sizes. An increase in the price of firm 1 decreases the relative utility of customers of firm 1 not only due to its direct effect, but also via a decrease in the expected network size of firm 1 and an increase in the expected network size of firm 2.

To avoid the possibility of cornered market equilibrium, we will adopt the assumption that the transportation cost parameter is greater than the relevant network benefits parameter; that is:

Assumption 1. $t>\beta$.

If this assumption does not hold, a consumer located closest to firm 2 may still be willing to buy from firm 1 at equal prices if she expects everyone else to do so. This raises the possibility of multiple consistent network sizes for given prices. In Section 4, we relax Assumption 1 by considering the case without product differentiation. Since $b_H>b_L$, Assumption 1 also implies $t>b_L>(1-\lambda)b_L$, a property we use repeatedly.

Firm i obtains profits of

$$\pi_i=(p_i-f)N_i.$$

Substituting (3) into profits for $i=1, 2$, taking the first-order conditions and solving out for prices implies equilibrium prices of

$$p_N^*\equiv p_1^*=p_2^*=f+t-\beta.$$

These are the same prices that arise in a model in which all consumers have network benefits parameter equal to β. Equilibrium prices are lower than the usual Hotelling prices due to the increased price sensitivity of demand under network effects.[8] Given that the firms equally share the market, the corresponding equilibrium profits for each firm equals

$$\pi_N^*=\frac{t}{2}-\frac{\beta}{2},$$

which is positive given Assumption 1.

Aggregate welfare is defined as the weighted sum of the consumers' and the firms' surpluses. Since unit demands are assumed, the possibility that higher prices lower welfare is captured by allowing the possibility that firms' surplus is discounted relative to consumer surplus. Let the weight on producer surplus in aggregate welfare be α where $0\leq\alpha\leq1$. Equilibrium welfare without multihoming is then

$$W_N^*=v+\frac{\beta}{2}-f-\frac{t}{4}-2(1-\alpha)\left(\frac{t}{2}-\frac{\beta}{2}\right).$$

[8] This consequence of network effects is known in the theoretical literature (Shy, 2001). It has also been confirmed in an experimental setting (Bayer and Chan, 2004).

The first two terms are the intrinsic and network benefits when there is no compatibility and no multihoming. The third and fourth terms represent the costs of providing the service and consumers' average transportation costs. In addition, to the extent the firms' surplus is discounted, welfare is decreasing in the firms' margins. Firms earn high margins if their products are more differentiated (transport costs are high) and low margins if network effects are strong (in which case firms compete aggressively in an attempt to capture the whole market).

2.2. Compatible firms

When firms are compatible, there are no product specific network effects as all consumers will be able to connect with one another regardless of which firm they join. This means $N_i=1$, and there is no reason to multihome. As network benefits offered by both firms are constant and equal, they cancel out in computing the location of the indifferent consumer, yielding the standard Hotelling share functions in both segments

$$s_1 = n_1 = \frac{1}{2} + \frac{p_2 - p_1}{2t},$$

and $n_2=s_2=1-n_1$.

We assume that achieving compatibility costs each firm a fixed amount F and is attained only when both firms undertake this investment. This ensures that there is no free riding problem in our framework since if compatibility raises profits, then each firm will be willing to incur the cost F given the rival firm also does. Thus, the firms will coordinate on the equilibrium with compatibility.[9]

The profit of firm i is therefore

$$\pi_i = (p_i - f)(\lambda s_i + (1-\lambda)n_i) - F.$$

Solving the first-order conditions, the corresponding equilibrium prices are the normal Hotelling equilibrium prices

$$p_C^* \equiv p_1^* = p_2^* = f + t.$$

Equilibrium profits are simply

$$\pi_C^* = \frac{t}{2} - F,$$

and equilibrium welfare is

$$W_C^* = v + \beta - f - \frac{t}{4} - 2F - 2(1-\alpha)\left(\frac{t}{2} - F\right).$$

In this case, welfare includes the maximal amount of network surplus. Relative to the no-compatibility and no-multihoming case, network benefits are doubled but welfare is lowered by the fixed costs of achieving compatibility and by higher prices (in the case that consumer surplus is valued more than producer surplus).

[9] Alternatively, we could have assumed only one firm has to incur the cost, but still obtained the same outcome by assuming the firms can first negotiate over whether to achieve compatibility and how to share costs.

2.3. The benchmark without multihoming

Given the symmetry of firms, their unilateral and joint incentives to make their services compatible are identical. For a given F, firms will make their services compatible only if there is an increase in equilibrium profits; that is, if π_C^* is higher than π_N^*, or equivalently if

$$F < \frac{\beta}{2} \equiv F_N^{\Pi}.$$

This condition trades off the higher margins to a firm resulting from compatibility (under compatibility, firms no longer compete to capture network effects) with the costs of reaching compatibility. Comparing W_N^* and W_C^*, a social planner will wish the services of the firms to be made compatible if

$$F < \frac{\beta}{2} - \frac{\beta}{4\alpha} \equiv F_N^W.$$

Whether the firms or the social planner will choose compatibility depends on the value of the fixed costs of achieving compatibility, F. Their incentives are aligned for both sufficiently low values and sufficiently high values of F. In contrast, for $F_N^W < F < F_N^{\Pi}$, the firms will choose compatibility even though this lowers welfare. For this range of costs, compatibility does not raise network benefits enough to cover the costs of achieving it. However, it raises industry profits more, reflecting that compatibility makes demands less price sensitive. This results in a transfer from consumers to firms, which increases profits but not welfare. We thus have:

Proposition 1. *Firms have an excessive incentive to choose compatibility in the absence of multihoming.*

This proposition is consistent with the results from the existing literature on network compatibility with price competition (such as those in Shy, 2001) and shows that these results extend to the case where consumers have heterogeneous valuations of network sizes. However, it is in contrast to the earlier results of Katz and Shapiro (1985) in which quantity setting homogenous-goods firms may have insufficient incentives to make their products compatible since they cannot capture the full surplus from doing so. In Section 4, we consider the case of homogenous products in our framework (by setting $t=0$), where like Katz and Shapiro we also find firms can have an insufficient incentive for compatibility in the absence of multihoming.

2.4. Incompatible firms with multihoming

In this section, we allow consumers to multihome, that is, to subscribe to both firms. We can immediately rule out some cases. First, there are no equilibria in which all consumers multihome, since then when faced with positive prices each individual consumer has no reason to multihome. Second, we are not interested in parameter values for which no consumer chooses to multihome in equilibrium, as in this case allowing consumers to multihome will not change the results. Given our assumption $b_L < \beta < t$, we can also rule out the case in which some low types multihome—the added transportation costs of them doing so always exceeds their gain in network benefits.[10] Instead, we will focus our analysis on an equilibrium where all low types singlehome (subscribe to one firm only) and all high types multihome. The only other possibility, in which *some* high types multihome while *all* low types singlehome, makes the analysis significantly more complicated without providing additional insights.

Formally, all high types multihoming and all low types singlehoming implies $s_i=1$ and $N_i=\lambda+(1-\lambda)n_i$ for $i=1, 2$. Subscribing to firm i exclusively allows a consumer to reach all high types and a share n_i of low types. As a result, the share of singlehoming consumers that join firm 1 is found by solving $U_1(n_1, b_L, N_1)=U_2(n_1, b_L, N_2)$ for n_1 which implies

$$n_1=\frac{1}{2}+\frac{p_2-p_1}{2(t-(1-\lambda)b_L)},$$

and $n_2=1-n_1$. Assumption 1 is sufficient to ensure the market share equation is well-behaved. Furthermore, it is straightforward to verify using the expression for n_1 that the total demand as well as the network size of firm i is given by

$$N_1=\frac{1}{2}+\frac{\lambda}{2}+\frac{(1-\lambda)(p_2-p_1)}{2(t-(1-\lambda)b_L)}. \tag{4}$$

Comparing the total demand faced by firm 1 without multihoming from (3) with that in (4), note that it shifts up by $\lambda/2$ and its price sensitivity changes. In particular, the total demand faced by firm 1 is less price sensitive when multihoming is allowed since

$$\frac{2(t-(1-\lambda)b_L)}{1-\lambda}-2(t-\lambda b_H-(1-\lambda)b_L)=\frac{2\lambda(t+(b_H-b_L)(1-\lambda))}{1-\lambda}\geq 0.$$

The reduction in price sensitivity combined with the upward shift in the demand implies that firms will charge higher prices when multihoming is allowed.

The profits of firm i are

$$\pi_i=(p_i-f)N_i.$$

Substituting the share function into profits for $i=1, 2$, taking the first-order conditions, solving out for prices, and simplifying implies candidate equilibrium prices of

$$p_M^*\equiv p_1^*=p_2^*=f+\frac{1+\lambda}{1-\lambda}(t-(1-\lambda)b_L). \tag{5}$$

Given Assumption 1, candidate equilibrium prices exceed costs f. In a technical Appendix to the paper, we show that there is a set of parameters for which prices given in (5)

[10] A similar logic explains why we need heterogeneous valuations of network sizes to obtain multihoming as an equilibrium outcome. Suppose all consumers obtain the same network benefits, say $b<t$, and prices are non-negative, then a necessary condition for multihoming is that $tx<b(1-N_1)$ and $t(1-x)<b(1-N_2)$. With some multihoming, $N_1>0$, $N_2>0$, and $N_1+N_2>1$, so that there is no $x\in[0,1]$ for which these conditions can all hold.

constitute a pure strategy equilibrium.[11] The existence of such an equilibrium requires b_H to be sufficiently large relative to b_L, but not too large, and most importantly that f is not too high.

In setting its price, each firm trades off the benefits of a higher price on its installed base of customers with a lower market share of singlehoming consumers. The existence of singlehoming consumers disciplines the prices that firms can charge, forcing them to take into account the usual competitive pressures on at least a portion of their customers. The market share of low types is still more sensitive to price than in a normal Hotelling framework because of the network benefits that arise from attracting additional low types (assuming $b_L > 0$). However, it is less sensitive to price than the case without multihoming. This reflects the fact that with multihoming, high types can be reached regardless of which firm low types subscribe to, reducing the impact of network effects on the pricing behavior of the firms. Moreover, the number of high types attracted does not depend on prices at the margin. We therefore get:[12]

Proposition 2. *Equilibrium prices and profits are higher when consumers are able to multihome compared to when they are not.*

Firms earn higher profits when more consumers multihome, both through higher prices as a result of the reduction in the price elasticity of demand, and through greater demand compared to the case without multihoming. An implication of this result is that whenever the firms can decide in a prior stage whether to require exclusivity from their customers (say as a technology choice, or through the use of an exclusive contract), they will not want to make their services exclusive. If either firm forces consumers to choose exclusively between itself or its rival, then price competition will be more intense and the firms will face lower demand. That firms prefer to allow multihoming, together with the possibility that it is often difficult (or perhaps illegal)[13] to impose such exclusivity, further justifies our focus on the equilibrium with multihoming.

Equilibrium welfare under multihoming is then

$$W_M^* = v + \lambda b_H + \left(1 - \lambda^2\right)\frac{b_L}{2} - (1 + \lambda)f - (1 + 3\lambda)\frac{t}{4} - 2(1 - \alpha) \times \frac{(1 + \lambda)^2}{2}\left(\frac{t}{1 - \lambda} b_L\right).$$

The first three terms measure the intrinsic and network benefits. The network benefits obtained by low types are now higher than without multihoming, reflecting the ability of low types to reach high types that multihome.[14] The fourth and fifth

[11] This Appendix is available on the journal website.

[12] The proof of the proposition, as with all subsequent ones, is given in the Appendix.

[13] One can imagine PlayStation only dealing with game developers who write software exclusively for their platform, or Visa only signing up merchants which do not accept rival cards, but it is difficult to imagine how firms can enforce exclusivity on consumers.

[14] The network benefits obtained by the $1 - \lambda$ low types are equal to b_L, a fraction λ of the time (since they can reach all high types) and are equal to $b_L/2$, a fraction $1 - \lambda$ of the time (since they can only reach the low types that are subscribed to the same firm). Then $(1 - \lambda)\ (\lambda + (1 - \lambda)/2)b_L = (1 - \lambda^2)b_L/2$.

terms represent the costs of providing the service and consumers' average transportation costs. Note that the costs of providing the service includes some duplication of subscription costs to the extent there is multihoming (so costs equal $2f$ a fraction λ of the time, and equal f a fraction $1-\lambda$ of the time). Similarly, transportation costs equal t a fraction λ of the time and equal $t/4$ a fraction $1-\lambda$ of the time. In addition, to the extent that the firms' profits are discounted relative to consumer surplus ($\alpha<1$), multihoming lowers welfare by raising prices and transferring surplus to firms. This is reflected in the last term.

Given the result in Proposition 2, that firms will prefer not to make their services exclusive, it is interesting to evaluate the welfare implications of this outcome. Comparing welfare with and without multihoming, we get that:

Proposition 3. *Assume a multihoming equilibrium exists. Then there exists an $\hat{\alpha}\in[0,1]$ such that welfare is higher under multihoming ($W_M \geq W_N$) whenever the social planner values profits sufficiently ($\alpha \geq \hat{\alpha}$) and welfare is higher without multihoming ($W_M < W_N$) whenever the social planner puts sufficiently low weight on profits ($\alpha < \hat{\alpha}$).*

For high values of α, the social planner values firms' profits similarly to consumer surplus, so that the increased prices are seen largely as a transfer from consumers to firms. In this case, the presence of multihoming raises overall welfare. Consumer surplus increases, reflecting that high types choose to multihome, which also confers a positive externality on low types. Moreover, firms are also better off (Proposition 2). On the other hand, multihoming has a negative impact on competitiveness, which obviously decreases consumer surplus, so that when the social planner is sufficiently concerned about consumer surplus it will prefer the case with exclusivity.

2.5. Private versus social compatibility decisions

When multihoming is possible, the incentives for compatibility become more complex. Given that consumers can multihome, firms will prefer to make their networks compatible if

$$F < \frac{(1+\lambda)^2 b_{\mathrm{L}}}{2} - \frac{\lambda(3+\lambda)t}{2(1-\lambda)} \equiv F_M^{\Pi}.$$

Clearly, when $b_{\mathrm{L}}=0$, so that low types get no network benefits, the firms will never make their services compatible when consumers can instead multihome (at least, for non-negative costs of achieving compatibility). In this case, the price sensitivity of demand with multihoming is lower than its level under compatibility. With multihoming, the demand of high types does not depend on price at the margin, while the demand of low types has the same sensitivity to price as in the case of compatibility. Furthermore, given the additional upwards shift in the demand due to double purchase by high type consumers, firms are able to sustain higher prices and earn higher profits than they can when they are compatible. Thus, even if there is no cost of achieving compatibility, firms will prefer to remain incompatible.

Comparing W_M^* with W_C^*, a social planner will wish the networks to be made compatible if

$$F < \frac{\frac{(1-\lambda)^2 b_{\mathrm{L}}}{2} + \lambda f + \frac{3\lambda t}{4} + (1-\alpha)\left(\frac{(1+\lambda)^2}{1-\lambda}(t-(1-\lambda)b_{\mathrm{L}})-t\right)}{2\alpha} \equiv F_M^W.$$

Compared to the case without multihoming, there are a number of new effects. Incompatibility now implies duplicated costs of customers subscribing to both firms (the term λf) and added transportation costs as customers travel to both firms (the term $3\lambda t/4$), neither of which affect the firms' compatibility decision. On the other hand, the losses in network benefits due to incompatibility are now less than before since consumers capture some of these benefits through multihoming anyway. Where the firms' surplus is weighted less than consumer surplus, compatibility also leads to an increase in welfare to the extent that prices are higher under the multihoming equilibrium. With a much reduced private incentive for compatibility and a generally increased social incentive for compatibility, for a range of parameter values firms will have an insufficient incentive to choose compatibility.

Proposition 4. *There exists $\bar{b}_L$ such that (i) whenever $b_L < \bar{b}_L$, firms have an insufficient incentive to choose compatibility (that is, $F_M^\Pi < F_M^W$). (ii) When $b_L > \bar{b}_L$, firms have an excessive incentive to choose compatibility (that is, $F_M^W < F_M^\Pi$); and (iii) if in addition $\alpha < \hat{\alpha}$ defined in Proposition 3, this excessive incentive of firms to choose compatibility is reduced as a result of the ability of consumers to multihome (that is, $F_N^W < F_M^W < F_M^\Pi < F_N^\Pi$).*

Proposition 4 shows that the possibility of multihoming may completely overturn the excessive tendency towards compatibility obtained for the benchmark case without multihoming. For instance, when the network benefits of low types are sufficiently low, we have already noted that firms will prefer incompatibility even if there are no costs to achieve compatibility. This means that there are times when a social planner will prefer compatibility although firms will not (but the reverse is not true).

For $b_{\mathrm{L}} > \bar{b}_L$, however, firms still have an excessive incentive towards compatibility, as in our benchmark model (that is, $F_M^W < F_M^\Pi$). Nevertheless, the firms' excessive tendency towards compatibility is unambiguously reduced as a result of multihoming when the social planner places a sufficiently low weight on the profits of the firms relative to consumer surplus. In this case, we find that $F_N^W < F_M^W < F_M^\Pi < F_N^\Pi$, so that for any fixed costs F where firms choose compatibility inefficiently under multihoming, they will do the same in the absence of multihoming (but the reverse is not true). In these cases, multihoming reduces the problem of firms choosing compatibility when they should not.

Regardless of parameter values, we can show that $F_M^\Pi - F_M^W < F_N^\Pi - F_N^W$. In particular, whenever $b_{\mathrm{L}} > \bar{b}$, we have $0 < F_M^\Pi - F_M^W < F_N^\Pi - F_N^W$. This result implies that the range of fixed costs where social and private incentives for achieving compatibility diverge is reduced. Clearly, this is a weaker statement than that in Proposition 4 as it allows the possibility of some fixed costs F for which multihoming results in firms choosing compatibility inefficiently where they will not otherwise. Nevertheless, it implies:

Proposition 5. *Whenever $b_L > \bar{b}_L$, the ability of consumers to multihome means that there is a smaller range of fixed costs of achieving compatibility for which firms have an excessive incentive towards compatibility.*

Whether Proposition 5 can be interpreted as a reduced tendency for firms to choose excess compatibility depends on how one interprets F. Since we wish to consider the incentives for compatibility over a range of possible values of F, suppose F is randomly drawn from the uniform distribution. Then, the probability that a compatibility problem exists is proportional to the range of values of F for which private and social incentives are not aligned. In this case, Proposition 5 implies that there is a smaller likelihood of fixed costs F arising for which firms have an excessive incentive towards compatibility as a result of consumers' ability to multihome.[15]

The effects of multihoming on private and social incentives to achieve compatibility can be explained intuitively as follows. First, take the case without multihoming. There are two opposing effects. The first effect comes from assuming prices are fixed. With fixed prices, firms do not profit from compatibility since they each still serve half of all customers, although compatibility raises network benefits as well as welfare. Thus, firms would have no incentive to choose compatibility even though welfare increases by $\beta/2$. The second effect comes from the fact firms will charge higher prices in the presence of compatibility, given that demand is less elastic. The higher prices have no impact on welfare (at least when $\alpha = 1$), while they increase industry profit by β. Comparing the two effects, the increase in profits is higher than the increase in welfare with compatibility, implying a higher incentive for firms to choose compatibility than the social planner.

Now consider how things change when we allow for multihoming. The first effect becomes less important, since some additional network benefits are now realized even without compatibility. The second effect also becomes less important, since firms will price higher even in the absence of compatibility due to the decreased strength of network effects as a result of multihoming. The net effect is to reduce the excessive incentive to choose compatibility.[16] In addition, there are a number of new effects that arise from high types multihoming. First, each firm faces higher demand from high types that buy from both firms. This raises the firms' profit directly, and also indirectly by causing them to set higher prices given that each faces a greater installed base of customers. This increase in profit is a transfer from consumers to firms, providing firms with too much incentive to avoid compatibility. In addition, compatibility now results in two new welfare gains which firms fail to internalize. First, there is the saving of the duplication in costs λf that would otherwise arise from multihoming consumers. Firms just price to recover their own costs, so that each firm ignores additional costs to society of $\lambda f/2$ under multihoming. Likewise, there is the saving of the duplication in transportation costs $3\lambda t/4$ that arise from

[15] For other distributions for F, this statement needs not be true.

[16] The closer b_L is to zero, the greater the reduction in the excessive incentive to choose compatibility as a result of multihoming. To see this, consider the case with $b_L = 0$. Then all network benefits are realized both with and without compatibility, so the first and second effects vanish.

multihoming consumers which firms ignore. For instance, these costs can include the costs of consumers having to deal with two different firms or the costs of having to learn to use two different services. Both of these effects provide further reasons why compatibility is socially desirable even though firms prefer incompatibility.

Somewhat paradoxically, the ability of consumers to multihome means policymakers may need to be more concerned about compatibility. Our results imply that this is more likely to be the case when network benefits are lower for low-value consumers, when there are more high-value (multihoming) consumers, when transportation costs are higher and when the cost of providing the service to each user is higher. This suggests that policymakers may have a reason to be concerned about firms which do not make their networks compatible in situations in which some consumers choose to multihome, where this multihoming involves significant duplication of costs, where this multihoming occurs through consumers subscribing to a second firm's service that is distant from their preferred choice and where some consumers put little value on network benefits. On the other hand, when firms do choose compatibility, our results suggest that policymakers should be less concerned that such compatibility is actually inefficient (or being used to reduce competition) given the ability of consumers to multihome.

2.6. An application to instant messaging

A recent public issue where our results could have been of use is related to the merger between AOL and Time Warner, which the FCC approved subject to conditions in January 2001. One area put under scrutiny was instant messaging (IM) services provided by AOL.[17] These services allow users to exchange messages with members who are present in a special directory. If competitors' directories are incompatible, then IM services exhibit firm specific network effects. A main concern of the FCC was that in the presence of these network effects, AOL's resistance to being compatible with competing IM providers, combined with the assets of Time Warner allowing high-speed data transmission, would mean that AOL's dominance of IM would translate to dominance in the market for advanced IM-based high-speed services.

Initially the FCC was of the view that provided the industry was one in which operators had similar subscriber bases, then firms would have sufficient incentives to achieve compatibility.[18] At the time, competitors of AOL's IM, Microsoft and Yahoo! engaged in significant lobbying activity to get FCC involved in the imposition of compatibility. AOL (2000) argued that any incompatibilities between different services are mitigated by means of consumers subscribing to multiple IM services simultaneously (multihoming), a view the FCC rejected. In fact, the order explicitly states "...We find the ability of users to use several IM services is not a substitute for interoperability..."[19] In their arguments, the FCC

[17] The discussion we present here is based on official and public documents that can be found at the FCC website (http://www.fcc.gov/mb/aoltw/aoltw.html). In particular, we use the order conditionally approving the merger (FCC, 2000), the second order which relieved AOL and Time Warner from the conditions set in the first order (FCC, 2003) as well as documents submitted by AOL (AOL, 2000) and experts (Faulhaber and Farber, 2003).

[18] FCC (2000), paragraph 154, p. 67.

[19] FCC (2000), paragraph 164, pp. 71–72.

deemed multihoming as an inconvenient solution, imposing a wide variety of costs on consumers. They interpreted the widespread multihoming as a result of high value placed by consumers on these services rather than an indication of relatively easy adoption. In contrast, compatibility was a matter of developing simple software applications and protocols. On these grounds, FCC approved the merger with the condition that the merged company could not provide advanced IM-based services until demonstrating that it had achieved interoperability with its competitors or a sufficiently competitive environment had emerged.

AOL petitioned in April 2003, documenting that competitors had gained significant market share and asked for relief from the requirements imposed by the FCC. Despite warnings of caution, most notably by two advisors to the FCC staff during the merger review (see Faulhaber and Farber, 2003), the FCC granted the company relief in August 2003. The FCC contended that there was sufficient competition in the market place. In answering comments, FCC (2003) suggested that the fact that AOL's competitors have been gaining market share at the expense of AOL's IM services, and that many consumers had subscribed to multiple IM platforms indicated that market tipping to one platform was not likely. The FCC also noted that Microsoft and Yahoo! were no longer lobbying for interoperability.

Our model suggests a different interpretation.[20] Given product differentiation in IM services, tipping may not have been the central concern to determine whether compatibility should be imposed. Instead, our analysis suggests that even if firms have comparable market shares and consumers multihome in order to reap maximal network benefits, this does not automatically mean that the issue of compatibility is redundant. In fact, the subsequent lack of support for interoperability by the major IM providers is consistent with the predictions of our model. Even though AOL and Microsoft had agreed on a framework to achieve interoperability, shortly after FCC's decision to remove the restrictions on the merger in August 2003, both companies abandoned their efforts in realizing compatibility.[21]

3. Two-sided markets

As noted in the Introduction, most of the applications where multihoming arises in practice involve two-sided markets (payment cards, entertainment platforms, and so on). In this section, we show that our model and findings can be extended to a two-sided market context. The model differs from the one presented above in that there are two

[20] To apply our model to this case requires some reinterpretation of our results given that IM programs are generally made available free of charge. A possible modelling approach is to assume that the companies receive a certain payoff per user, and that they use IM as a way to attract these users. The strategic variable firms can control is then the value of the IM services provided, which can be analyzed by using the competition in utilities framework of Armstrong and Vickers (2001). Using this approach, we have verified our main results remain valid qualitatively.

[21] "AOL and Microsoft drop idea to connect IM services," New Media Age, 16 October 2003, reported by Wendy Mcauliffe.

groups of users that can be distinguished by the side of the market they belong to. Specifically, we consider two groups of agents A and B, each of which consists of high and low types with $\lambda_A = \lambda_B = \lambda$, just as in our one-sided model. Each group has identical preferences to those in the one-sided case, except that each group values the number of agents belonging to the *other* group, but not the number of agents within the same group. Consistent with the literature on two-sided markets, we consider two platforms which are able to set different prices to the two different groups. Finally, to make results comparable to the one-sided market case, we assume that it costs each platform F to make itself compatible with the rival's platform for each group of agents (that is, in total each platform has to incur $2F$ to achieve compatibility). With these assumptions, we get:

Proposition 6. *Profits and welfare under compatibility, incompatibility and multihoming in a symmetric two-sided market are equal to two equivalent one-sided markets.*

The result demonstrates that there is nothing intrinsic about two-sidedness that causes results to change one way or another. The effects identified in the one-sided case remain. In particular, multihoming can result in insufficient incentives for platforms to achieve compatibility in a two-sided market setting. Once the two sides of the market are allowed to be asymmetric, the results are no longer identical to before although the same basic findings can be obtained.[22]

To understand the underlying mechanism leading to Proposition 6, we present the case without multihoming. Exactly the same type of argument can be used for the case with multihoming, which is given in the Proof of Proposition 6 in the Appendix. For the case with compatibility the result follows directly given that network benefits no longer play any role.

In the absence of either multihoming or compatibility, demand functions facing each platform are now different in a two-sided market compared to equivalent one-sided markets. Previously, demand was given in (3). In the case of a two-sided market, using superscripts to denote each side of the market, demands are determined by

$$N_1^A = \frac{1}{2} + \frac{t(p_2^A - p_1^A) + \beta(p_2^B - p_1^B)}{2(t^2 - \beta^2)}$$

$$N_1^B = \frac{1}{2} + \frac{t(p_2^B - p_1^B) + \beta(p_2^A - p_1^A)}{2(t^2 - \beta^2)},$$

with $N_2^A = 1 - N_1^A$ and $N_2^B = 1 - N_1^B$. Given $t > \beta$, demand for group A is more sensitive to the price charged to group A than the price charged to group B and the demand for group B is more sensitive to the price charged to group B than the price charged to group A. This implies that the price sensitivity of each group's demand will be smaller in magnitude in a two-sided market compared to an equivalent one-sided market.[23] A price change on one

[22] A separate Appendix, available on the journal website, analyzes this case.
[23] The derivative is now $-1/2(t - \beta^2/t)$, whereas before it was $-1/2(t - \beta)$. Given $t > \beta$, the first expression is smaller in magnitude than the second.

side of the market only induces an indirect feedback effect in a two-sided market, given that the price on the other side is fixed. On the other hand, given that profits are obtained from both groups, each platform now takes into account that any increase in price on one side of the market will reduce demand on the other side.

To see how these two effects combine, note that the profit function of platform i is

$$\pi_i = \left(p_i^{\mathrm{A}} - f\right)N_i^{\mathrm{A}} + \left(p_i^{\mathrm{B}} - f\right)N_i^{\mathrm{B}}.$$

Choosing the profit-maximizing prices for platform i, and exploiting symmetry between the two platforms, we can express the equilibrium price on each side of the market as

$$p_i^{\mathrm{A}} = f + t - \beta + \beta\left(1 - \frac{\beta}{t}\right) - \frac{\beta}{t}\left(p_i^{\mathrm{B}} - f\right)$$

$$p_i^{\mathrm{B}} = f + t - \beta + \beta\left(1 - \frac{\beta}{t}\right) - \frac{\beta}{t}\left(p_i^{\mathrm{A}} - f\right).$$

Comparing these prices to the prices ($p_N^* = f + t - \beta$) in the one-sided market case, the first additional term measures the extent to which prices are now higher due to decreased price elasticity, while the second additional term measures the extent to which prices are now lower due to the cross-market externality effect. Solving the two equations simultaneously it is clear that the two additional terms exactly cancel each other out. The result is that platforms will set identical prices to those in the equivalent one-sided market, so that equilibrium profit for each platform is $2\pi_N^*$ and welfare is $2W_N^*$, where these expressions are defined in Section 2.1. This analysis shows that there are offsetting effects when one moves from one-sided to two-sided markets and it is not necessarily the case that one particular effect will dominate the other.

4. Strong network benefits

So far our results have been derived in situations where the effect of product differentiation is assumed to be stronger than that of network benefits. In this section, we explore what can happen if this assumption is reversed by considering the special case in which there is no product differentiation. It is known that in the absence of product differentiation firms will generally have insufficient incentives to make themselves compatible.[24] We are interested in how the possibility of multihoming affects this compatibility problem.

One possibility is that firms are able to choose (in a prior stage) whether to allow multihoming or whether to make their services exclusive. With pure network benefits, a dominant firm (enjoying favorable beliefs) will prefer exclusivity, in which case

[24] See, for instance, Katz and Shapiro (1985) for the quantity competition case.

multihoming is not an option that has to be considered in the analysis of compatibility. This provides one justification for our focus on the case with product differentiation.

Of course, it may not be feasible for firms to stop consumers from multihoming. Even so, the possibility of multihoming can still be irrelevant. In a one-sided market setting with price competition it is natural to consider the case where all consumers buy from a single (dominant) firm. For instance, this outcome arises if beliefs stubbornly favor one firm over another (say the incumbent over the entrant). A rival firm that undercuts cannot profitably attract demand since consumers will not join a firm at a positive price if they do not expect any other consumers to do so. In this case, allowing consumers to multihome may be an irrelevant option, as consumers get maximal network benefits buying from the dominant firm and so continue to coordinate on this firm. The dominant firm will be able to exploit network effects to obtain maximal profits. In contrast, compatibility results in pricing at cost. The dominant firm will therefore oppose compatibility, as will the entrant if there are any costs of achieving compatibility. A social planner concerned with total surplus will also oppose compatibility if there are any costs of achieving compatibility, since with one firm taking the whole market the full network benefits will be realized. The possibility of multihoming does not affect this result since multihoming never arises in this setting. On the other hand, if the social planner prefers lower prices, it may favor a solution with compatibility, but the dominant firm will not sponsor any such move. Then, the ability of consumers to multihome does not solve (or change) the compatibility problem that arises when there are strong network effects.

The fact that the dominant firm will block compatibility when network effects are strong and multihoming is not allowed also applies in a two-sided market context. To see this, consider setting transportation costs and intrinsic benefits to zero in the model of Section 3 but allowing for asymmetry. Suppose group A consists entirely of high types ($\lambda_A=1$) and group B consists entirely of low types ($\lambda_B=0$). Define the network benefits to each group as b_A and b_B, such that $b_A=b_H$, $b_B=b_L$ and $b_A>b_B$. Finally, assume that the policymaker is only concerned with total surplus ($\alpha=1$), which ensures that in the absence of multihoming both the platforms and the policymaker prefer incompatibility for any positive cost of achieving compatibility.

Without compatibility or multihoming, the dominant platform will set prices of b_A to group A and b_B to group B. If agents coordinate on the dominant platform, it is impossible for a rival platform to profitably attract agents away.[25] The dominant platform will then make maximal profits of $b_A+b_B-f_A-f_B$, where we have allowed for different costs on each side of the market. Since the platform extracts all the surplus, total surplus just equals the platform's profit. The result is equivalent to the one-sided case above. The dominant platform will oppose compatibility, which results in pricing at cost, as will the entrant and the social planner given any costs of achieving compatibility.

Once we allow for multihoming, our model corresponds to that of chicken-and-egg competition in Caillaud and Jullien (2003, Section 5) in the special case where matching in

[25] This is not true if agents can multihome, as then a rival platform can bribe one group to multihome with a slightly negative price and then attract the other group by slightly undercutting the dominant platform's price (a divide-and-conquer strategy).

their model is perfect. This allows us to make use of their results. Caillaud and Jullien characterize the maximal profit multihoming equilibria in their Proposition 11. Group A users are charged $b_A/2$, having all their surplus extracted, while group B users are just charged cost f_B. Group A multihomes, while group B splits between the two platforms. Assuming $b_A/2 \geq f_A$, this gives the platforms a non-negative profit of $b_A/2 - f_A$ each. Welfare will be $b_A + b_B - 2f_A - f_B$ reflecting the duplication of costs given group A multihomes. Platforms will both resist compatibility, which results in pricing at cost. Total surplus under compatibility is $b_A + b_B - f_A - f_B - 2F$, so the social planner prefers compatibility if $2F < f_A$.

Summarizing these results, we have that platforms prefer to remain incompatible regardless of whether users multihome or not, while the social planner's preference for compatibility depends on whether users multihome or not. When the costs of compatibility are not too high compared to the cost of duplication, the social planner will prefer compatibility to the multihoming outcome, although platforms will never invest to achieve it. Thus, we have:[26]

Proposition 7. *In a two-sided market setting without product differentiation, and in which the policymaker weights consumer and producer surplus equally, platforms have the correct incentive to choose compatibility in the absence of multihoming but an insufficient incentive to choose compatibility in the presence of multihoming.*

The conclusion then is that in a one-sided setting, multihoming does not create a compatibility problem since multihoming never arises in equilibrium. On the other hand, in a two-sided setting, multihoming can arise in equilibrium and as a result it can create a compatibility problem where one previously did not exist.

5. Conclusions

The point of this paper is to illustrate that just because some consumers achieve network benefits by subscribing to multiple networks does not mean that policymakers can ignore the issue of compatibility of networks. In the cases we looked at, the ability of consumers to multihome generally made it more likely (not less) that firms will block compatibility when compatibility is efficient. By the same token, our results suggest policymakers can be more relaxed in the case firms do choose to become compatible. In the presence of multihoming, firms are less likely to choose compatibility when it is inefficient to do so.

Our findings have implications for a wide range of industries including various communication services, hardware/software standards, payment networks, and entertainment systems. We discussed the case of instant messaging platforms in Section 2.6. As another example, consider payment schemes, such as those offered by American Express,

[26] When the social planner values consumer surplus more than profits ($\alpha < 1$), there is a compatibility problem both with and without multihoming. However, for α sufficiently close to 1, multihoming results in a larger range of fixed costs where platforms do not become compatible even though it is socially desirable.

MasterCard and Visa. Many consumers now hold multiple such cards and many merchants accept more than one card. Such multihoming may suggest that from a social point of view there is no reason to consider the case for forcing these networks to make their networks compatible. Cardholders can choose to hold a single payment card and yet have little problem using their card at merchants. Likewise, merchants seldom cannot make a sale because consumers lack the right card. However, multihoming can be costly, both directly in terms of duplicated costs and the added inconvenience for users, but also indirectly because it can weaken competition between the platforms providing these services, making them less likely to prefer compatibility when it would otherwise be desirable. Thus, asking whether such networks should be made compatible is an interesting question, even in the presence of such widespread multihoming.

Appendix A

A.1. Proof of Proposition 2

The difference in equilibrium prices caused by multihoming is

$$p_M^* - p_N^* = \frac{1+\lambda}{1-\lambda}(t-(1-\lambda)b_{\mathrm{L}}) - t + \beta$$
$$= \left(\frac{1+\lambda}{1-\lambda} - 1\right)(t-(1-\lambda)b_{\mathrm{L}}) + \lambda b_{\mathrm{H}} > 0.$$

The corresponding equilibrium profits under multihoming are

$$\pi_M^* = \frac{(1+\lambda)^2}{2}\left(\frac{t}{1-\lambda} - b_{\mathrm{L}}\right),$$

which is positive given Assumption 1. Comparing equilibrium profits with those without multihoming,

$$\pi_M^* - \pi_N^* = \frac{(1+\lambda)^2}{2}\left(\frac{t}{1-\lambda} - b_{\mathrm{L}}\right) - \left(\frac{t}{2} - \frac{\lambda b_{\mathrm{H}} + (1-\lambda)b_{\mathrm{L}}}{2}\right)$$
$$= \frac{1}{2}\left(\frac{(1+\lambda)^2}{1-\lambda} - 1\right)(t-(1-\lambda)b_{\mathrm{L}}) + \frac{\lambda b_{\mathrm{H}}}{2} > 0. \quad \square$$

A.2. Proof of Proposition 3

Let us parameterize the welfare function with α and denote it by $W_i(\alpha)$, $i=\{M, N\}$. Observe that $W_M(\alpha) - W_N(\alpha)$ is increasing in α, since

$$\frac{\partial}{\partial\alpha}[W_M(\alpha) - W_N(\alpha)] = \lambda b_{\mathrm{H}} + \frac{(3+\lambda)\lambda(t-(1-\lambda)b_{\mathrm{L}})}{1-\lambda} = 2(\pi_M^* - \pi_N^*) > 0,$$

where the final inequality follows from Proposition 2.

Setting $\alpha = 1$ in the difference of welfare expressions with and without multihoming, we obtain

$$W_M(1) - W_N(1) = \frac{1}{2}\lambda b_{\mathrm{H}} + \frac{1}{2}\lambda(1-\lambda)b_{\mathrm{L}} - \lambda f - \frac{3}{4}\lambda t,$$

which is positive whenever

$$f \leq \frac{1}{2}b_{\mathrm{H}} + \frac{1}{2}(1-\lambda)b_{\mathrm{L}} - \frac{3}{4}t \equiv x_4.$$

In a technical Appendix to the paper, available at the journal's website, we show that high types multihome in equilibrium if and only if

$$f < \frac{(1-\lambda)b_{\mathrm{H}}}{2} + (1+\lambda)b_{\mathrm{L}} - \frac{2t}{1-\lambda} \equiv x_0.$$

Therefore, this is a necessary condition for the multihoming equilibrium we focus on to exist. Using these definitions, we have

$$x_4 - x_0 = \frac{1}{2}\frac{3\lambda^2 b_{\mathrm{L}} + t}{1-\lambda} + \frac{1}{2}\lambda b_{\mathrm{H}} + \frac{1}{2}\frac{t - b_{\mathrm{L}}}{1-\lambda} + \frac{3}{4}\frac{\lambda(t - b_{\mathrm{L}})}{1-\lambda} + \frac{1}{4}\frac{t - \lambda b_{\mathrm{L}}}{1-\lambda} > 0,$$

and hence $x_4 > x_0$, yielding $W_M(1) > W_N(1)$.

Similarly, setting $\alpha = 0$ in the difference of welfare expressions with and without multihoming yields

$$W_M(0) - W_N(0) = -\frac{\lambda}{2b_{\mathrm{H}}} - \lambda f - \frac{7\lambda(t-(1-\lambda)b_{\mathrm{L}})}{2(1-\lambda)} - \frac{\lambda(t-\lambda(1-\lambda)b_{\mathrm{L}})}{4(1-\lambda)} - \frac{\lambda^2(t-(1-\lambda)b_{\mathrm{L}})}{4(1-\lambda)} < 0,$$

implying $W_M(0) < W_N(0)$. Therefore, there must exist an $\hat{\alpha} \in [0, 1]$ such that $W_M(\hat{\alpha}) = W_N(\hat{\alpha})$. Thus, for $\alpha \geq \hat{\alpha}$, we have $W_M(\alpha) \geq W_N(\alpha)$, while $W_M(\alpha) < W_N(\alpha)$, for $\alpha < \hat{\alpha}$. □

A.3. Proof of Proposition 4

We have

$$F_M^{\Pi} - F_M^{W} = \frac{2\left(1 + 5\lambda - 5\lambda^2 - \lambda^3\right)b_{\mathrm{L}} - 4\lambda(1-\lambda)f - \lambda(15+\lambda)t}{8(1-\lambda)\alpha} = \frac{\left(1 + 6\lambda + \lambda^2\right)b_{\mathrm{L}}}{4\alpha} - \frac{\lambda f}{2\alpha} - \frac{\lambda(15+\lambda)t}{8(1-\lambda)\alpha},$$

which is negative whenever

$$b_{\mathrm{L}} < \frac{2\lambda(1-\lambda)f + \lambda(15+\lambda)t}{(1-\lambda)\left(1 + 6\lambda + \lambda^2\right)} \equiv \bar{b}_{\mathrm{L}}.$$

This implies $F_M^{\Pi} < F_M^{W}$, proving the result in (i) and (ii).

On the other hand, for $b_L > \bar{b}_L$ we have $F_M^W < F_M^\Pi$, while $\alpha < \hat{\alpha}$ implies $W_M < W_N$ from Proposition 3, so that $F_N^W < F_M^W$. From Proposition 2 we have $\pi_M^* > \pi_N^*$, so that $F_M^\Pi < F_N^\Pi$. Combining these results gives $F_N^W < F_M^W < F_M^\Pi < F_N^\Pi$, which implies a reduction in the excessive tendency towards compatibility proving (iii) in the proposition. □

A.4. Proof of Proposition 5

Regardless of the value of $b_L < t$, we have

$$F_M^\Pi - F_M^W - \left[F_N^\Pi - F_N^W\right] = -\frac{\lambda\left(12\lambda b_L + 2\lambda^2 b_L + 2(1-\lambda)b_H + 15t - 14b_L + 4(1-\lambda)f + \lambda t\right)}{8(1-\lambda)\alpha} < 0.$$

In particular, when $b_L > \bar{b}_L$, both with and without multihoming firms have excessive incentives towards compatibility. However, the range of fixed costs where this excessive incentive occurs is reduced with multihoming. □

A.5. Proof of Proposition 6

When the platforms are made compatible, the prices will just be the normal Hotelling prices $f+t$ as consumers get the same (maximal) network benefits irrespective of the platform they join. Network benefits, regardless of whether they arise from agents from the same group (in the one-sided case), or across groups (in the two-sided case), drop out of the demand functions. Facing equal prices, each group will divide equally between the two platforms, so $N_1^A = N_1^B = 1/2$. As a result, equilibrium profit for each firm is $2\pi_C^*$ and welfare is $2W_C^*$, where these expressions are defined in Section 2.2.

The logic for the multihoming case follows exactly from the case of incompatibility and no multihoming presented in Section 3. With multihoming, previously demand was given in (4). In the two-sided market case, demands are determined by

$$N_1^A = \frac{1}{2} + \frac{\lambda}{2} + \frac{(1-\lambda)t\left(p_2^A - p_1^A\right) + (1-\lambda)^2 b_L\left(p_2^B - p_1^B\right)}{2\left(t^2 - (1-\lambda)^2 b_L^2\right)}$$

$$N_1^B = \frac{1}{2} + \frac{\lambda}{2} + \frac{(1-\lambda)t\left(p_2^B - p_1^B\right) + (1-\lambda)^2 b_L\left(p_2^A - p_1^A\right)}{2\left(t^2 - (1-\lambda)^2 b_L^2\right)},$$

with $N_2^A = 1 - N_1^A$ and $N_2^B = 1 - N_1^B$. Using these demand functions, and proceeding as before, the equilibrium price on each side of the market can be written as

$$p_i^A = f + \frac{(1+\lambda)t}{(1-\lambda)} - (1+\lambda)b_L + (1+\lambda)\left(1 - \frac{(1-\lambda)b_L}{t}\right)b_L - \frac{(1-\lambda)b_L\left(p_i^B - f\right)}{t}$$

$$p_i^B = f + \frac{(1+\lambda)t}{(1-\lambda)} - (1+\lambda)b_L + (1+\lambda)\left(1 - \frac{(1-\lambda)b_L}{t}\right)b_L - \frac{(1-\lambda)b_L\left(p_i^A - f\right)}{t}.$$

Solving the two equations simultaneously shows that platforms will set identical prices to those in the one-sided market equivalent, so that equilibrium profit for each firm is $2\pi_M^*$ and welfare is $2W_M^*$, where these expressions are defined in Section 2.4.

Appendix B. Supplementary material

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.ijindorg.2005.07.004.

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