Strategic level three-stage production distribution planning with capacity expansion

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Abstract

In this paper, we address a strategic planning problem for a three-stage production-distribution network. The problem under consideration is a single-item, multi-supplier, multi-producer and multi-distributor production-distribution network with deterministic demand. The objective is to minimize the costs associated with production, transportation and inventory as well as capacity expansion costs over a given time horizon. The limitations are the production capacities of the suppliers and producers, and transportation capacities of the corresponding transportation network. On the other hand, all capacities may be increased at a fixed cost. The problem is formulated as a 0-1 mixed integer programming model. Since the problem is intractable for real life cases efficient relaxation-based heuristics are considered to obtain a good feasible solution.

Keywords: Production-distribution planning, mixed integer programming, heuristics
1. Introduction

In the last decades, competitive pressures pose the challenge of simultaneously prioritizing the dimensions of competition: flexibility, cost, quality and delivery. In addition to these dimensions, other factors such as the speed with which products are designed, manufactured and distributed, as well as the need for higher efficiency and lower operational costs, are forcing companies to continuously search for ways to improve their operations. Firms are using optimization models and algorithms, decision support systems and computerized analysis tools to improve their operational performance and remain competitive under the threat of increasing competition.

A production-distribution system is referred to as an integrated system consisting of various entities that work together in an effort to acquire raw materials, convert these raw materials into specified final products and deliver these final products to markets (Beamon, 1998). Production part of these systems includes design and management of the entire manufacturing process. Distribution and logistics part determines how the products are retrieved and transported from warehouses to retailers.

In this paper, a single-item, multi-supplier, multi-producer and multi-distributor production-distribution network is formulated as a mixed integer programming model. Main limitations of the problem are capacity constraints on the supplier and producer; however these capacities can be expanded with a fixed cost. The objective is to minimize the costs associated with production, transportation and inventory as well as capacity expansion costs over a given time horizon. The problem is formulated as a 0-1 mixed integer programming model and three common sense heuristics are developed in an attempt to obtain good solutions in a reasonable amount of time.

The remainder of the paper is organized as follows: Section 2 discusses the relevant literature. Section 3 introduces the mathematical model and Section 4 presents the solution
approach. The experimental design is described in Section 5 and the results are analyzed in Section 6. Finally, the conclusion and directions for future research are given in Section 7.

2. Literature Review

Production-distribution planning is one of the most important activities in supply chain management (SCM). To implement SCM in real logistic world, supply chains have been modeled in analytical ways using deterministic or stochastic methods.

There is a vast amount of articles on the integrated production-distribution literature. Although classification of related literature is hard due to the wide variety of assumptions and multiplicity in objective functions, a general classification is possible. The design of the distribution system and production planning processes may be classified as strategic level work, as the optimization problems on a given production-distribution system is considered as tactical level work. In line with the scope of the paper, the literature review presented here contains mathematical programming models on integrated production-distribution. However, interested readers may refer to Vidal and Goetschalckx (1997), Beamon (1998) and Erengüç et al. (1999) for a detailed literature review on models and methods for integrated production-distributions systems.

One of the early works on the topic dates back to the work of Geoffrion and Graves (1974) which presents an algorithm based on Benders Decomposition to solve a multi-commodity single-period production-distribution problem. The contribution of the paper is the method of solution, which converges to the solution in a few iterations for a specified difference between upper and lower bounds.

Geoffrion et al. (1978) present a status report in strategic distribution system planning based on decomposition techniques. Geoffrion et al. (1982) present a final
version of this paper with a more thorough description of the system and more managerial emphasis, but with the same model as in their former research.

Williams (1981) proposes seven heuristics for a deterministic joint production-distribution scheduling problem. The objective of each heuristic is to determine the production-distribution schedule which satisfies final demand while minimizing the average inventory holding and fixed costs associated with ordering and processing.

Hodder and Dinçer (1986) are first to include financial considerations caused by the international facility location decisions. Exchange rates, subsidized financing, preferential tax treatments, market prices and international interest rates are implicitly included in the objective function. The authors use a multifactor approach in order to transform large-scale quadratic MIP into a more tractable model. They report solutions for 1600 continuous and 20 integer variables based on two approaches.

Cohen and Lee (1988) present a comprehensive model on linking the decisions between different entities of the supply chain and improving their performance by using stochastic demand data. The structure of the model consists of several sub-models each representing different part of a supply chain. Cohen and Moon (1991) present a MIP model to determine product line assignments as well as determining raw material requirements, production volumes and shipments. They apply an algorithm and report solutions for the small problems with 60 binary variables and 204 continuous variables in 49 seconds of CPU time.

Arntzen et al. (1995) include multinational considerations in their optimization problem. The objective is to optimize the global supply chain of Digital Equipment Corporation. They report solutions to problems of 6000 constraints, few hundreds of binary variables by non-traditional methods. However exact solution method is not provided in the paper.
A real life application is presented by Brown et al. (2001) for Kellogg Company. The authors propose two approaches to the problem. First approach is solving the model in weekly detail and second approach is planning the production-distribution network in monthly time periods in order to make capacity expansion and consolidation decisions. The tactical version of the problem is solved with a heuristic called sliding time window.

Barbarosoglu and Özgür (1999) propose a Lagrangean relaxation based solution procedure. They attempt to decouple the system with relaxation and use subgradient optimization to facilitate the information flow between sub-problems. The main contribution of this study is the forward algorithm applied to distribution sub-problem.

Jang et al. (2002) present a supply network design and production-distribution planning problem and attempt to solve it by splitting it into modules. Genetic algorithm (GA) is used as solution methodology. Small-scale examples are solved using CPLEX 6.5 for comparison. The authors report 0.2% gap between the GA solutions and solutions from CPLEX.

Yan et al. (2003) add logical constraints to the production-distribution problem. Their main contribution is adding BOM limitations as logical constraints to the mixed integer representation of the problem. One small-scale problem result is presented to show solution validity.

The aim of this paper is to model the strategic level production-inventory-transportation planning problem of a three stage system as a 0-1 MIP problem and to propose three linear programming relaxation based heuristics to obtain good solutions fast. In what follows is the formulation of the mathematical model.
3. Model Formulation

Our case represents a system consisting of first-tier suppliers, main production plants and distribution centers. Deterministic demand is considered and demand points are distribution centers. The model is designed as a capacitated, multi-facility, single-item production-distribution system.

Capacity limitation on suppliers, producers and corresponding transportation network can be expanded with a fixed cost. After capacity expansion, due to contractual costs, variable production costs also changes. Inventory holding is allowed only at the producer stage.

In the design of model, we make the following assumptions:

- Demand is deterministic.
- Backlogging is not allowed.
- There is a variable transportation cost between the supplier and producer, and the producer and distribution center.
- Production capacities at the supplier and producer are limited but can be expanded with a fixed cost and increased variable cost per unit.
- Transportation capacities between the supplier and producer and the producer and distributor are limited but can be expanded with a fixed cost and variable cost per unit of increased capacity.
- Investment decisions to increase the capacity are made at the beginning of each period and are not carried to next periods.
- Distribution and manufacturing lead times are negligible.
- Only the producer may hold inventory without any capacity limitation.
- Fixed costs are associated with the outsourcing of transportation, like contractual costs arising from carrying additional quantities.
Demand at every stage is satisfied on just-in-time basis (JIT).
- There is a 1:1 ratio between raw materials and finished goods.

The parameters and decision variables of the model are as follows:

**Parameters**

- \( p_{it} \): Amount of raw material cost per unit at supplier \( i \) in period \( t \)
- \( m_{jt} \): Amount of production cost per unit at producer \( j \) in period \( t \)
- \( R_t \): Available total transportation capacity from suppliers to producers in period \( t \)
- \( S_t \): Available total transportation capacity from producers to distributor \( k \) in period \( t \)
- \( G_i \): Available production capacity at supplier \( i \)
- \( C_j \): Available production capacity at producer \( j \)
- \( A_{ijt} \): Fixed cost for transportation capacity increase between supplier \( i \) and producer \( j \) in period \( t \)
- \( B_{jkt} \): Fixed cost for transportation capacity increase between producer \( j \) and distributor \( k \) in period \( t \)
- \( E_{it} \): Fixed cost for production capacity increase in supplier \( i \) in period \( t \)
- \( e_{it} \): Variable cost for per unit production capacity increase in supplier \( i \) in period \( t \)
- \( F_{jt} \): Fixed cost for production capacity increase in producer \( j \) in period \( t \)
- \( f_{jt} \): Variable cost per unit production capacity increase in producer \( j \) in period \( t \)
- \( trs_{ijt} \): Transportation cost per unit between supplier \( i \) and producer \( j \) in period \( t \)
- \( trp_{jkt} \): Transportation cost per unit between producer \( j \) and distributor \( k \) in period \( t \)
- \( h_{jt} \): Unit inventory cost in producer \( j \) in period \( t \)
- \( d_k \): Demand at distributor \( k \) in period \( t \)
- \( a \): Discount rate (0.2 %)
**Decision variables**

- $x_{ijt}$: Raw material shipped from supplier $i$ to producer $j$ in period $t$
- $y_{jkt}$: Product shipped from producer $j$ to distributor $k$ in period $t$
- $I_{jt}$: Inventory at producer $j$ at the end of period $t$
- $u_{ijt}$: Added transportation capacity from supplier $i$ to producer $j$ in period $t$
- $n_{jt}$: Added production capacity at producer $j$ in period $t$
- $w_{it}$: Added supply capacity of supplier $i$ in period $t$
- $v_{jkt}$: Added transportation capacity from producer $j$ to distributor $k$ in period $t$

\[

U_{\alpha} = \begin{cases} 
1 & \text{if } u_{\alpha} > 0 \\
0 & \text{otherwise}, 
\end{cases}
\]

\[
V_{\alpha} = \begin{cases} 
1 & \text{if } v_{\alpha} > 0 \\
0 & \text{otherwise}, 
\end{cases}
\]

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W_{\alpha} = \begin{cases} 
1 & \text{if } w_{\alpha} > 0 \\
0 & \text{otherwise}, 
\end{cases}
\]

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N_{\alpha} = \begin{cases} 
1 & \text{if } n_{\alpha} > 0 \\
0 & \text{otherwise}. 
\end{cases}
\]

The system under consideration consists of multi-suppliers which provide raw materials to multiple production plants producing a single item distributed to several distribution centers. Here, the distribution centers are operated by wholesale companies operating independently. The multi-echelon nature with fixed costs both in the production and transportation activities complicates the problem and it becomes difficult to find an efficient procedure to solve the resulting formulation to optimality. The problem is formulated as a 0-1 mixed integer program as follows:
Min \[
\sum_{i \in T} \sum_{s \in S} \sum_{j \in P} p_{ij} x_{ij} + \sum_{i \in T} \sum_{j \in P} \sum_{k \in D} m_{jk} v_{jk} + \sum_{i \in T} \sum_{j \in P} (e_{ij} W_{ij} + E_{ij} W_{ij}) + \sum_{j \in P} \sum_{s \in S} \sum_{k \in D} (f_{jk} n_{jk} + F_{jk} N_{jk}) + \sum_{i \in T} \sum_{j \in P} \sum_{s \in S} \sum_{k \in D} (trp_{ijk} v_{jkt} + B_{jk} V_{jkt}) + \sum_{i \in T} \sum_{j \in P} l_{ij} h_{ij} + \sum_{j \in P} \sum_{s \in S} \sum_{k \in D} \sum_{l \in D} (trp_{jkl} y_{jkt})
\]

s.t.
\[
\sum_{j \in P} \sum_{k \in D} x_{ij} - \sum_{j \in P} \sum_{k \in D} u_{ij} \leq R_i \quad \forall i, j, t
\]
\[
\sum_{j \in P} \sum_{k \in D} y_{jkt} - \sum_{j \in P} \sum_{k \in D} v_{jkt} \leq S_k \quad \forall j, k, t
\]
\[
\sum_{j \in P} x_{ij} \leq G_i + w_{it} \quad \forall i, t
\]
\[
\sum_{i \in S} x_{ij} \leq C_j + n_{jt} \quad \forall j, t
\]
\[
I_{j-1} + \sum_{i \in T} x_{ij} - I_{jt} = \sum_{k \in D} y_{jkt} \quad \forall j, t
\]
\[
\sum_{j \in P} y_{jkt} = d_{kt} \quad \forall k, t
\]
\[
u_{ij} \leq Z_{ij} U_{ij} \quad \forall i, j, t
\]
\[
v_{jkt} \leq Z_{jkt} V_{jkt} \quad \forall j, k, t
\]
\[
w_{it} \leq Z_{it} W_{it} \quad \forall i, t
\]
\[
n_{jt} \leq Z_{jt} N_{jt} \quad \forall j, t
\]
\[
U_{ij}, V_{jkt}, W_{it}, N_{jt} \in \{0,1\}
\]

The objective (1) is to minimize costs associated with the production, transportation, inventory holding and capacity expansion. Constraints (2) are the transportation capacity constraints ensuring that the total raw materials shipped from supplier \(i\) to producer \(j\) in period \(t\) does not exceed the total available capacity and the expanded capacity of routes \((i-j)\). Constraints (3) are the similar transportation capacity constraints for the routes between producers and distributors. Constraints (4) are the supply capacity constraints for supplier \(i\) and provide that raw materials shipped from supplier \(i\) to producer \(j\) in period \(t\) should not exceed the supply capacity of supplier \(i\) and its expanded
capacity. Constraints (5) are the production capacity constraint at the producer. The inventory balance constraints are expressed in constraints (6). Constraints (7) are demand constraints which state that total products shipped from all producers to distributor $k$ in period $t$ should exactly match the demand of distributor $k$ in period $t$. Constraints (8-11) are binary constraints for capacity expansions. Constraints (8) and (9) are for transportation capacity expansion between supplier $i$ and producer $j$ in period and between producer $j$ and distributor $k$ in period $t$, respectively. Constraints (9) and (10) are additional supply and production capacity constraints, respectively. All $Z$s are sufficiently large scalars satisfying the capacity increases.

4. Solution Methodology

Integrated models with centralized planning naturally lead to complex, large-scale models which are difficult to solve optimally in most real-life cases. Hence, it becomes a necessity to develop alternative solution techniques which are able to provide near optimal solutions for all organizational divisions in the integrated model (Barbarosoglu and Özgür, 1999). Among many methods used for solving this kind of intractable problems, decomposition and heuristics are shown to perform well.

There are two basic kinds of heuristic approaches that can be designed. First is heuristics that are based on optimization theory and aims to accelerate or truncate optimization method, such as partial branch and bound method used in Maes et al (1991). Other heuristics are common sense heuristics based on intuition or common rules applied to a problem (Maes et al., 1991).

This paper proposes three simple linear programming (LP)-based heuristics to obtain good solutions in a reasonable time. Common sense heuristics proposed in this
study try to achieve cost savings by eliminating fixed costs. In what follows is the
description of each heuristic approach.

**LP Heuristic 1**

LP Heuristic 1 (LPH1) starts with the LP relaxation solution of the problem. After
obtaining the relaxation solution, it finds the largest non-integer binary variable, forces it to
1 by adding a constraint and resolves the problem. This process continues until all binary
variables are equal to 1 or 0. LPH1 is depicted in Figure 1.

*** INSERT Figure 1 ***

**LP Heuristic 2**

LP Heuristic 2 (LPH2) which is also based on LP-relaxation aims to achieve cost
reductions by evaluating the trade-off between holding inventory instead of expanding
capacity and incurring the fixed cost. Since holding inventory is possible at only producer
stage, heuristic starts with first two production binary variables of the highest fixed cost
producer. The algorithm first checks two consecutive time period capacity expansion
variables together. If the sum of consecutive binary variables equals 1, second binary
variable is forced to 0 and the other is forced to 1 by adding two constraints to the problem,
that is next period’s demand is produced in the current period and carried in inventory for
one period. After resolving this LP, there can be two consequences: new solution can be
infeasible or objective function does not improve. In this case last two constraints are
deleted from the problem, largest variable is forced to 1 and the problem is resolved.
Otherwise, if there is an improvement in the objective function, heuristic continues with
checking the next two consecutive binaries.
If, at the beginning, there are no two variables such that their sum is 1, first non-integer variable is forced to 1 or 0 depending on whether it is greater or less than 0.5. The heuristic stops when all binary variables are 1 or 0. The detailed description is provided in Figure 2.

*** INSERT Figure 2 ***

**LP Heuristic 3**

LP Heuristic 3 (LPH3) is based on LPH2. Contribution is heuristic’s ability to check the tightest capacity level and improving solution based on that capacitated stage. In our case, most capacitated level is the largest fixed cost and the minimum capacity producer. By this prescreening feature, the heuristic tries to make big improvements at the beginning and aim to save time.

5. **Experimental Study**

Small-scale examples with 534 continuous and 282 binary variables and large-scale examples with 2976 continuous and 1536 binary variables are considered for experimental tests. These data sets are characterized by the number of suppliers, producers, distributors and the length of time horizon. For exact comparison, first run of experiments are conducted with a small example consisting of 5 suppliers, 3 producers and 8 distributors over a planning horizon of 6 months. The small-scale examples may be solved to optimality within a reasonable computational time using ILOG CPLEX Concert Technology 2.0 and allow us to make a sound comparison. Still, a time limit of 300 seconds is imposed for the sake of time management in case of tight capacity examples
which may require longer computational time. Other data set consists of 8 suppliers, 5 producers, 15 distributors and analysis horizon is 12 months.

To accurately reflect the effect of capacity, fixed and variable costs, different cases are evaluated in the data sets. First of all, production and transportation capacities are set to 60% of total demand in tight capacity case. In loose capacity case capacities are set to 90% of total demand. Raw material cost is set to 10, and production cost is determined as 5% and 20% of raw material cost and interpreted as added value at the production plant. Extra supply and production costs are set to 10% of production cost, which is total raw material cost and manufacturing value added. Inventory cost per item/day is 2% of production cost.

All data is generated according to uniform distribution. The demand data comes from U(50,500). Transportation costs between the supplier and producer are generated using U(0.5, 1.5) and U(0.5, 3.5) for low and high transportation costs, respectively. Transportation costs between producer and distributor comes from U(0.6, 1.80) and U(0.6, 4.20) for low and high transportation costs, respectively.

In computational tests, the effect of fixed cost is investigated by choosing fixed cost 10 times and 100 times greater than the average production costs. Transportation fixed cost is chosen to be the 100 times and 500 times the average transportation cost. As a result 1024 sample problems from the each data set is generated with C++. For each set of parameters 5 problems of small type and 3 problems of large type are solved. In total, 8192 problems are solved.

6. Results and Analysis

All three of the heuristics are coded in C++ and solved on a PC with 2.00 GHz Xeon processor. Branch-and-cut method of CPLEX Concert Technology 2.0 is used for benchmarking. The results in the tables are grouped into four categories with respect to
their transportation cost combination (high-low) and production cost combination (high-low). The legend can be found in Table 1.

Some general observations may be made regarding the small problem setting. First of all, when more than two of capacity restrictions are tight, CPLEX may not solve the problem to optimality in 300 seconds. However, heuristics provide very close solutions compared to the optimal (only good feasible in some cases) solutions obtained by CPLEX in a few seconds using LPH1 or LPH2. It is worth noting that as the capacities become looser solution quality of heuristics deteriorate and CPLEX can reach the optimal solution in a few seconds. The problems with tight capacity and high fixed costs for all entities (i.e. supplier, producer, transportation network between supplier-producer and producer-distributor) cannot be solved to optimality in 300 seconds. In total, 91.5% of 5120 small problems are solved to optimality.

*** INSERT Table 1 ***

Specifically, if low transportation cost alternative is chosen, LPH2 performs better than other heuristics. Solution time of LPH2 is less than that of LPH3 and more than that of LPH1. Another observation is that, regardless of the production and fixed costs, the solution quality of LPH2 gradually decreases as capacity constraints loosen, thus heuristics have no advantages over CPLEX. The reason is that; there is generally no need for capacity expansion in problems with two or more loose capacity in entities, which means there are only a few non-integer variables in the LP relaxation solution. The improvements obtained using the heuristics which are based on rounding non-integer variables will become insignificant in such cases. Average errors in solutions for low transportation-low
production cost and low transportation-high production cost cases may be found in Figure 3 and Figure 4, respectively.

*** INSERT Figure 3 and 4 ***

When the transportation cost is high, LPH2 still gives better solutions regardless of the level of production cost (Refer to Figures 5 and 6).

*** INSERT Figure 5 and 6 ***

For large-scale examples, a time limit of 150 seconds is imposed in CPLEX. It is observed that LPH1 performs better than other heuristics. In low transportation cost and low production cost case LPH1 produces good results except for two problem sets: FKH-FIL-FPH-FSH and FKL-FIH-FPH-FSH. Percent errors vary between 0.4-0.6% of CPLEX solutions (Note that CPLEX was able to find the optimal solution in 3 problem instances out of 3072 problems). For LPH1, solution times are 7.15 seconds on the average, 6.97 seconds in the best and 7.36 seconds in the worst case. Solution times for LPH2 are 12.90 seconds on the average, 12.64 in the best and 13.3 in the worst case. Solution times for LPH3 are 12.74 in the average, 12.12 in the best and 13.10 in the worst case. Even when the transportation cost is high LPH1 performs better than other heuristics with all fixed cost cases and 0.51% deviation in the average is achieved compared to CPLEX solutions. CPLEX solutions are obtained in 159.38 seconds in the average. Detailed results for TL-PL and TH-PL cases may be found in Figure 7 and Figure 8, respectively.

*** INSERT Figure 7 and 8 ***
Same performance is observed with the high production cost problems regardless of the transportation cost. LPH1 gives good feasible solutions in 7.17 seconds in the average, 6.97 in the best and 7.36 in the worst case. However, it should be noted that CPLEX takes much larger time to give a feasible solution (158.86 seconds in the average, 150 seconds in the best and 291.11 seconds in the worst case). As the capacity restrictions loosen solution times for heuristics increase, solution times for CPLEX decrease. Detailed results may be found in Figure 9 and Figure 10.

*** INSERT Figure 9 and 10 ***

7. Conclusion and Future Work

This paper proposes a mathematical formulation of a multi-period three-stage strategic production-distribution planning problem and presents a simple and fast methodology to solve this problem. The proposed model includes the links between entities and this integrated approach provides an understanding of the minimization of system-wide costs which include production, inventory and transportation costs as well as costs associated with the increase in production and transportation capacities and in supply quantities.

Three heuristics are developed based on the LP relaxation solution of the problem. The efficiency of the heuristics is tested with an extensive computational study. We conclude that heuristics provide good feasible solutions for complex problems with little computational effort compared to the feasible solutions obtained using CPLEX with significantly longer computational times. Even if CPLEX provides optimal solutions in a reasonable time (which is the case in only 3 problem instances in a total of 3072 large-scale problems), heuristic codes may still be preferable since they are easy to use generic
codes and accessible to everyone while CPLEX is a licensed program which requires skills to use.

The model presented assumes that the capacity increases are contract based and does not allow carrying the additional capacities to the subsequent periods. However, the increase in capacities may be permanent in the case of one-time investments for acquisition of land, building, machinery and/or logistics components. Thus, the performance of the heuristics for this case may be explored in the future.

Since demand fluctuations are more common in real life situations, a stochastic modeling approach may also be addressed. Service level requirements may be incorporated within the stochastic demand case.

In addition, the performance of CPLEX may be improved by feeding the solution obtained using the heuristics as an initial feasible solution. The significance of the improvement in the solution quality and computational effort using the heuristic solution as compared to starting from scratch will be investigated in the future.

References


Step 0: Solve LP Relaxation.
Step 1: Select the largest non-integer capacity expansion binary variable and round it to 1.
Step 2: Resolve the LP. If solution is all integer STOP, else go to Step 1.

Figure 1. Description of LP Heuristic 1

Step 0: Solve LP relaxation.
Step 1: From the production capacity expansion non-integer variables of the most expensive producer, check if there exists two consecutive $j_1$ and $j_2$ such that $j_1 + j_2 = 1$ and $j_1$ and $j_2$ are as small as possible. If exists go to Step 2, otherwise go to Step 4.
Step 2: Force $N_{j_1}$ to 1 and $N_{j_2}$ to 0. Resolve LP.
Step 3: If LP solution is infeasible or if there is no improvement in objective function, then delete the most recently added constraint which forced $N_{j_2}$ to 0 and resolve LP. Go to Step 1.
Step 4: If $j_1 > 0.5$ force $N_{j_1}$ to 1, otherwise to 0. Resolve LP.
Step 5: If infeasible, replace the most recent added constraint to 1 and resolve LP. Go to Step 1.

Figure 2. Description of LP Heuristic 2

Figure 3. Percent Errors for TL-PH case for small problems
Figure 4. Percent Errors for TL-PL case for small problems

Figure 5. Percent Errors for TH-PL case for small problems
Figure 6. Percent Errors for TH-PH case for small problems

Figure 7. Percent Errors for TL-PL case for large problems
Figure 8. Percent Errors for TH-PL case for large problems

Figure 9. Percent Errors for TL-PH case for large problems
Figure 10. Percent Errors for TH-PH case for large problems

Table 1. Legend

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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>distributor</td>
<td>producer</td>
<td></td>
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</tbody>
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